What You Should Already Know
by Dr. Colton, Physics 581 (last updated: Fall 2020)

Chemistry

- The Periodic Table: valence electrons, atomic mass
- Atomic orbitals: what s, p, d, and f refer to
- Spin degeneracy: two electrons in each orbital (spin up and spin down)
- Molar mass:
  - number of moles = \( \frac{m}{MM} \) ("\( MM \)" = molar mass; make sure units of \( m \) and \( MM \) agree)
  - number of atoms = number of moles \( \times N_A \) = \( \frac{m}{MM} \times N_A \), where \( N_A \) = Avogadro’s number
- Hydrogen atom energy levels: \( E_n = -\frac{13.6 \text{ eV}}{n^2} \); 13.6 eV is known as the Rydberg constant

Physics 121

- Vectors, typically symbolized by bold letters
  - How to add/subtract
  - How to do dot and cross products
  - Unit vector notation, e.g. \( \hat{\mathbf{x}} = (1,0,0) \), etc.
- Newton’s 2nd Law: \( \Sigma \mathbf{F} = m \mathbf{a} \)
- Energy, divided into…
  - Kinetic energy of object: \( KE = \frac{1}{2}mv^2 \)
  - Potential (for “conservative” forces), often symbol \( U \)
- Momentum of object: \( p = mv \)
- Connection between kinetic energy and momentum: \( KE = \frac{p^2}{2m} \)
- Springs
  - Force by spring: \( F = -kx \) (Hooke’s law; \( k \) is the “spring constant”)
  - Potential energy stored in spring: \( PE = \frac{1}{2}kx^2 \)
- SI Units: distance measured in meters, force in newtons, mass in kilograms, and energy in joules

Physics 123

- Thermodynamics:
  - \( R = \frac{k_B}{N_A} \) (relationship between universal gas constant \( R \), Boltzmann’s constant \( k_B \), and Avogadro’s number \( N_A \))
  - Heat, symbol \( Q \)
    - Heat transfer by thermal conduction: \( \frac{\Delta Q}{\Delta t} = \frac{k\Delta T}{l} \), where \( k \) = thermal conductivity
    - Specific heat \( c \) defined by: \( Q = mc\Delta T \)
    - Molar heat capacity \( C \) defined by: \( Q = nC\Delta T \)
- General wave properties:
  - What all of these parameters mean in the equations below: \( x, t, A, \lambda, f, T, v, k, \omega, \phi \)
  - Plane wave: \( f = A \cos(kx - \omega t + \phi) \leftrightarrow \) (complex representation) \( f = A e^{i(kx - \omega t + \phi)} \)
• Complex representation may be new to you; there’s an implied “take the real part” to both sides of the equation; it follows from Euler’s identity (see below).
  o How those parameters interrelate:
    • wavenumber $k = \frac{2\pi}{\lambda}$
    • angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$
    • wave speed $v = \lambda f = \frac{\omega}{k}$
  o Group velocity: $v_{group} = \frac{d\omega}{dk}$ (this equation may be new to you)
  o Waves on a string:
    • $v = \sqrt{\frac{T}{\mu}}$, where $T =$ tension and $\mu =$ linear mass density
    • Reflected power $R = \left(\frac{v_2-v_1}{v_2+v_1}\right)^2$ (this equation may be new to you)
• Electromagnetic waves:
  o Spectrum of light (approximate boundaries):
    • 10 - 400 nm = UV
    • 400 - 700 nm = visible
    • 700 nm - 100 μm = IR
  o Index of refraction, $n$
    • speed of light in a material $= \frac{c}{n}$
    • $n = \sqrt{\epsilon_r}$, where $\epsilon_r$ is the dielectric constant (sometimes symbol $K$), aka the “relative permittivity” (this fact may be new to you)
• Diffraction:
  o 2 slit result: $d \sin \theta_{bright} = m\lambda$, where $d =$ distance between slits
  o Bragg’s Law: $2d \sin \theta = m\lambda$, where $d =$ separation between crystal planes

Physics 220
• Electric fields
  o Force from electric field: $F = qE$, where $q$ is the charge (in coulombs) and $E$ is the electric field
  o Potential difference (in volts) $\Delta V = -\int_{path} E \cdot d\ell$, which means that $|E| = \frac{\Delta V}{d}$ in regions where electric field is constant
• Circuits
  o Ohm’s Law: $V = IR$, where $I$ is the current and $R$ is the resistance of the circuit element
  o Resistance of an ohmic material $R = \frac{\rho \ell}{A}$, where $\rho =$ resistivity, $\ell$ is the length, and $A$ is the cross-sectional area
• Magnetic fields:
  o Lorentz force: $F = qv \times B$, force on a moving charge; use right hand rule for direction
  o Hall effect: transverse voltage when a current is passed through a perpendicular magnetic field
• Coulomb’s Law:
  o $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$ (field from a point charge located at origin; $\frac{1}{4\pi\varepsilon_0}$ also sometimes written as $k_e$, the Coulomb force constant)
  o $U = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r}$ (potential energy of interaction between two charges $q$ and $Q$)

What you should already know – pg 2
Flux of a field through a surface: \( \Phi = \int_{\text{surface}} (\text{Field}) \cdot dA \)

Maxwell’s equations in integral form
- Gauss’s Law
  \[ \oint_{\text{closed surface}} \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \] (electric flux is proportional to \( q_{\text{enclosed}} \))
- Gauss’s Law for magnetism
  \[ \oint_{\text{closed surface}} \mathbf{B} \cdot d\mathbf{A} = 0 \] (no point sources of magnetic flux; “no magnetic monopoles”)
- Faraday’s Law
  \[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \] (changing magnetic fields act as sources of electric fields)
- The left hand side is the induced voltage or EMF (electromotive force), so this is also written as induced EMF = –d(magnetic flux)/dt
- Ampere’s Law, with Maxwell correction
  \[ \oint_{\text{closed path}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}} + \varepsilon_0 \mu_0 \frac{d\Phi_B}{dt} \] (currents act as sources of magnetic fields; so do changing electric fields)

Physics 222
- Photons
  - photon momentum \( p = \frac{h}{\lambda} \)
  - photon energy \( E = hf = h\omega = \frac{hc}{\lambda} = pc \)
- Quantum mechanical wavefunctions:
  - Schrödinger equation: \(-\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} + U(x)\Psi = E\Psi \) (1D, time independent)
    - \( U(x) \) = potential energy function; \( \Psi \) and \( E \) are the solutions to equation, namely the allowed wavefunctions and corresponding energies
  - \( \int_{x_1}^{x_2} |\Psi|^2 dx \) = probability of finding particle between \( x_1 \) and \( x_2 \)
  - \( \int_{-\infty}^{\infty} |\Psi|^2 dx = 1 \), normalization condition
- “Particle in a box” aka “infinite square well”, where particle restricted to region between \( x = 0 \) and \( x = L \)
  - Wavefunctions: \( \Psi_n = A \sin \left( \frac{n\pi x}{L} \right) \)
  - Energies: \( E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = n^2E_1 \)

Math
- Complex numbers
  - Complex numbers as points in the complex plane; polar ↔ rectangular conversion
  - Euler’s identity: \( e^{ix} = \cos x + i \sin x \)
- Basic calculus
  - Basic derivatives
  - Basic integrals

What you should already know – pg 3
Taylor series

\[ f(x_0 + \Delta x) = f(x_0) + \frac{df}{dx}\bigg|_{x=x_0}(\Delta x) + \frac{d^2f}{dx^2}\bigg|_{x=x_0}\frac{(\Delta x)^2}{2!} + \cdots \]  
(general formula)

\((1 + x)^n \approx 1 + nx\) for small \(x\) (useful specific application)

- Fourier series, when \(f(x)\) = periodic function with period \(L\) (\(x\) and \(L\) have units of meters)
  - Fundamental frequency \(k_0 = \frac{2\pi}{L}\) (this is called a “spatial frequency”, units of \(m^{-1}\))
  - Expansion in terms of multiples of the fundamental frequency:
    \[ f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nk_0x) + \sum_{n=1}^{\infty} b_n \sin(nk_0x) \]
    Coefficients given by:
    \[ a_0 = \frac{1}{L} \int_{0}^{L} f(x) \, dx, \text{ which is just the average value of the function} \]
    \[ a_n = \frac{2}{L} \int_{0}^{L} f(x) \cos(nk_0x) \, dx \]
    \[ b_n = \frac{2}{L} \int_{0}^{L} f(x) \sin(nk_0x) \, dx \]

  - Exponential version of Fourier series
    \[ f(x) = \sum_{n=-\infty}^{\infty} c_n e^{-ink_0x} \]
    \[ c_n = \frac{1}{L} \int_{0}^{L} f(x) e^{+ink_0x} \, dx \]

- Linear Algebra
  - How to solve simultaneous equations via matrices; here’s a 3 equation example where the \(a_{ij}\)’s and \(C\)’s are known numbers:
    \[
    \begin{align*}
    a_{11}x + a_{12}y + a_{13}z &= C_1 \\
    a_{21}x + a_{22}y + a_{23}z &= C_2 \\
    a_{31}x + a_{32}y + a_{33}z &= C_3
    \end{align*}
    \Rightarrow
    \begin{pmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
    \end{pmatrix}
    \begin{pmatrix}
    x \\
    y \\
    z
    \end{pmatrix}
    =
    \begin{pmatrix}
    C_1 \\
    C_2 \\
    C_3
    \end{pmatrix}
    \Rightarrow
    \begin{pmatrix}
    x \\
    y \\
    z
    \end{pmatrix}
    =
    \begin{pmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
    \end{pmatrix}^{-1}
    \begin{pmatrix}
    C_1 \\
    C_2 \\
    C_3
    \end{pmatrix}
    \]
  - Determinants: how to calculate, what significance is \((\det(M) = 0\) if and only if \(M\) has no inverse\)
  - How to solve eigenvalue-type equations using determinants; here’s a 3 equation example:
    \[
    \begin{pmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
    \end{pmatrix}
    \begin{pmatrix}
    x \\
    y \\
    z
    \end{pmatrix}
    = \lambda \begin{pmatrix}
    x \\
    y \\
    z
    \end{pmatrix}
    \text{ a common type of equation, called an “eigenvalue equation”}
    \]
    \[
    \begin{pmatrix}
    a_{11} - \lambda & a_{12} & a_{13} \\
    a_{21} & a_{22} - \lambda & a_{23} \\
    a_{31} & a_{32} & a_{33} - \lambda
    \end{pmatrix}
    \begin{pmatrix}
    x \\
    y \\
    z
    \end{pmatrix}
    = 0
    \Rightarrow
    \det
    \begin{pmatrix}
    a_{11} - \lambda & a_{12} & a_{13} \\
    a_{21} & a_{22} - \lambda & a_{23} \\
    a_{31} & a_{32} & a_{33} - \lambda
    \end{pmatrix}
    = 0
    \]
    The determinant = 0 condition gives an algebraic equation for \(\lambda\) from which you can get the 3 allowed values but programs such as Mathematica and Matlab will have functions to automatically find the eigenvalues of a matrix (i.e. solve the eigenvalue equation) without having to manually do the algebra.

- Multivariable Calculus
  - Scalar and vector functions:
    - Example: \(f(x, y, z) = x^2y + \sin z\) is a scalar function of \(x, y, z\).
    - Example: \(F(x, y, z) = (x^2y + \sin z, xyz, 4)\) is a vector function of \(x, y, z\).
    Equivalent statements are:
      - \(F(x, y, z) = (x^2y + \sin z)\hat{x} + xyz\hat{y} + 4\hat{z}\)
      - \(F_x = x^2y + \sin z, F_y = xyz, F_z = 4\)
Grad of a scalar function: \( \nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \), which is a vector function.

Divergence of a vector function: \( \nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \), which is a scalar function.

Curl of a vector function: \( \nabla \times \mathbf{F} = \left| \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{array} \right| \), which is a vector function.

Divergence theorem: \( \oint_{\text{closed}} \mathbf{F} \cdot d\mathbf{A} = \int_{\text{volume bounded}} \nabla \cdot F \, dV \)

Stokes' theorem: \( \oint_{\text{closed}} \mathbf{F} \cdot d\mathbf{l} = \int_{\text{surface bounded by the path}} (\nabla \times \mathbf{F}) \cdot d\mathbf{A} \)

Both of those last two theorems are summarized like this: the integral of a function over a closed boundary can be obtained by adding up (integrating) a derivative over the region of space being bounded.

**Computational Skills**

- Know how to use a program such as Mathematica to do the following:
  - Symbolic integrals
  - Symbolic algebra, if desired
  - Numeric integrals
  - Numeric root finding, including finding the intersection of two functions
  - Matrix inverses
  - Matrix multiplication
  - Matrix eigenvalues
  - Plotting

- Know how to do basic programming in a language of your choice