What You Should Already Know
by Dr. Colton, Physics 581 (last updated: Fall 2020)

Chemistry

- The Periodic Table: valence electrons, atomic mass
- Atomic orbitals: what s, p, d, and f refer to
- Spin degeneracy: two electrons in each orbital (spin up and spin down)
- Molar mass:
    - number of moles $= \frac{m}{MM}$ ("MM" = molar mass; make sure units of *m* and *MM* agree)
    - number of atoms $=$ number of moles $\times N_A = \frac{m}{MM} \times N_A$, where N_A = Avogadro's number
- Hydrogen atom energy levels: $E_n = -\frac{13.6\text{ eV}}{n^2}$; 13.6 eV is known as the Rydberg constant

Physics 121

- Vectors, typically symbolized by **bold** letters
    - How to add/subtract
    - How to do dot and cross products
    - Unit vector notation, e.g. $\hat{\mathbf{x}} = (1,0,0)$, etc.
- Newton's 2nd Law: $\Sigma\mathbf{F} = m\mathbf{a}$
- Energy, divided into…
    - Kinetic energy of object: $KE = \frac{1}{2}mv^2$
    - Potential (for "conservative" forces), often symbol U
- Momentum of object: $p = mv$
- Connection between kinetic energy and momentum: $KE = \frac{p^2}{2m}$
- Springs
    - Force by spring: $\mathbf{F} = -k\mathbf{x}$ (Hooke's law; k is the "spring constant")
    - Potential energy stored in spring: $PE = \frac{1}{2}kx^2$
- SI Units: distance measured in meters, force in newtons, mass in kilograms, and energy in joules

Physics 123

- Thermodynamics:
    - $R = \frac{k_B}{N_A}$ (relationship between universal gas constant R, Boltzmann's constant k_B, and Avogadro's number N_A)
    - Heat, symbol Q
        - Heat transfer by thermal conduction: $\frac{\Delta Q}{\Delta t} = \frac{kA\Delta T}{L}$, where *k* = thermal conductivity
        - Specific heat c defined by: $Q = mc\Delta T$
        - Molar heat capacity C defined by: $Q = nC\Delta T$
- General wave properties:
    - What all of these parameters mean in the equations below: $x, t, A, \lambda, f, T, v, k, \omega, \phi$
    - Plane wave: $f = A\cos(kx - \omega t + \phi)$ $\leftrightarrow$ (complex representation) $f = A\, e^{i(kx-\omega t+\phi)}$

        - Complex representation may be new to you; there's an implied "take the real part" to both sides of the equation; it follows from Euler's identity (see below).
    - How those parameters interrelate:
        - wavenumber $k = \frac{2\pi}{\lambda}$
        - angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$
        - wave speed $v = \lambda f = \frac{\omega}{k}$
    - Group velocity: $v_{group} = \frac{d\omega}{dk}$ (this equation may be new to you)
    - Waves on a string:
        - $v = \sqrt{\frac{T}{\mu}}$, where T = tension and μ = linear mass density
        - Reflected power $R = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)^2$ (this equation may be new to you)
- Electromagnetic waves:
    - Spectrum of light (approximate boundaries):
        - 10 - 400 nm = UV
        - 400 - 700 nm = visible
        - 700 nm - 100 μm = IR
    - Index of refraction, n
        - speed of light in a material = $\frac{c}{n}$
        - $n = \sqrt{\epsilon_r}$ where ϵ_r is the dielectric constant (sometimes symbol K), aka the "relative permittivity" (this fact may be new to you)
- Diffraction:
    - 2 slit result: $d \sin\theta_{bright} = m\lambda$, where d = distance between slits
    - Bragg's Law: $2d \sin\theta = m\lambda$, where d = separation between crystal planes

Physics 220

- Electric fields
    - Force from electric field: $\mathbf{F} = q\mathbf{E}$, where q is the charge (in coulombs) and E is the electric field
    - Potential difference (in volts) $\Delta V = -\int_{path} \mathbf{E} \cdot d\boldsymbol{\ell}$, which means that $|E| = \frac{\Delta V}{d}$ in regions where electric field is constant
- Circuits
    - Ohm's Law: $V = IR$, where I is the current and R is the resistance of the circuit element
    - Resistance of an ohmic material $R = \frac{\rho\ell}{A}$, where ρ = resistivity, ℓ is the length, and A is the cross-sectional area
- Magnetic fields:
    - Lorentz force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, force on a moving charge; use right hand rule for direction
    - Hall effect: transverse voltage when a current is passed through a perpendicular magnetic field
- Coulomb's Law:
    - $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$ (field from a point charge located at origin; $\frac{1}{4\pi\epsilon_0}$ also sometimes written as k_e, the Coulomb force constant)
    - $U = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r}$ (potential energy of interaction between two charges q and Q)

- Flux of a field through a surface: $\Phi = \int_{surface} (\mathbf{Field}) \cdot d\mathbf{A}$
- Maxwell's equations in integral form
  - Gauss's Law
    - $\oint_{\substack{closed \\ surface}} \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enclosed}}{\epsilon_0}$ (electric flux is proportional to $q_{enclosed}$)
  - Gauss's Law for magnetism
    - $\oint_{\substack{closed \\ surface}} \mathbf{B} \cdot d\mathbf{A} = 0$ (no point sources of magnetic flux; "no magnetic monopoles")
  - Faraday's Law
    - $\oint_{\substack{closed \\ path}} \mathbf{E} \cdot d\ell = -\frac{d\Phi_B}{dt}$ (changing magnetic fields act as sources of electric fields)
      - The left hand side is the induced voltage or EMF (electromotive force), so this is also written as induced EMF = –d(magnetic flux)/dt)
  - Ampere's Law, with Maxwell correction
    - $\oint_{\substack{closed \\ path}} \mathbf{B} \cdot d\ell = \mu_0 I_{enclosed} + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$ (currents act as sources of magnetic fields; so do changing electric fields)

Physics 222

- Photons
  - photon momentum $p = \frac{h}{\lambda}$
  - photon energy $E = hf = \hbar\omega = \frac{hc}{\lambda} = pc$
- Quantum mechanical wavefunctions:
  - Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + U(x)\Psi = E\Psi$ (1D, time independent)
    - $U(x)$ = potential energy function; Ψ and E = the solutions to equation, namely the allowed wavefunctions and corresponding energies
  - $\int_{x_1}^{x_2} |\Psi|^2 dx$ = probability of finding particle between x_1 and x_2
  - $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$, normalization condition
- "Particle in a box" aka "infinite square well", where particle restricted to region between $x = 0$ and $x = L$
  - Wavefunctions: $\Psi_n = A \sin\left(\frac{n\pi x}{L}\right)$
  - Energies: $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = n^2 E_1$

Math

- Complex numbers
  - Complex numbers as points in the complex plane; polar ↔ rectangular conversion
  - Euler's identity: $e^{ix} = \cos x + i \sin x$
- Basic calculus
  - Basic derivatives
  - Basic integrals

- Taylor series
  - $f(x_0 + \Delta x) = f(x_0) + \left.\frac{df}{dx}\right|_{x=x_0} (\Delta x) + \left.\frac{d^2 f}{dx^2}\right|_{x=x_0} \frac{(\Delta x)^2}{2!} + \cdots$ (general formula)
  - $(1+x)^n \approx 1 + nx$ for small x (useful specific application)

- Fourier series, when $f(x)$ = periodic function with period L (x and L have units of meters)
  - Fundamental frequency $k_0 = \frac{2\pi}{L}$ (this is called a "spatial frequency", units of m^{-1})
  - Expansion in terms of multiples of the fundamental frequency:
    - $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nk_0 x) + \sum_{n=1}^{\infty} b_n \sin(nk_0 x)$
  - Coefficients given by:
    - $a_0 = \frac{1}{L}\int_0^L f(x)dx$, which is just the average value of the function f
    - $a_n = \frac{2}{L}\int_0^L f(x)\cos(nk_0 x)\,dx$
    - $b_n = \frac{2}{L}\int_0^L f(x)\sin(nk_0 x)\,dx$
  - Exponential version of Fourier series
    - $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{-ink_0 x}$
    - $c_n = \frac{1}{L}\int_0^L f(x)e^{+ink_0 x}dx$

- Linear Algebra
  - How to solve simultaneous equations via matrices; here's a 3 equation example where the a_{ij}'s and C's are known numbers:

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= C_1 \\ a_{21}x + a_{22}y + a_{23}z &= C_2 \\ a_{31}x + a_{32}y + a_{33}z &= C_3 \end{aligned} \Rightarrow \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}^{-1}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

  - Determinants: how to calculate, what significance is (det(**M**) = 0 if and only if **M** has no inverse)
  - How to solve eigenvalue-type equations using determinants; here's a 3 equation example:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$ a common type of equation, called an "eigenvalue equation"

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= \lambda x \\ a_{21}x + a_{22}y + a_{23}z &= \lambda y \\ a_{31}x + a_{32}y + a_{33}z &= \lambda z \end{aligned} \Rightarrow \begin{pmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \Rightarrow \det\begin{pmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{pmatrix} = 0$$

The determinant = 0 condition gives an algebraic equation for λ from which you can get the 3 allowed values but programs such as Mathematica and Mathematica will have functions to automatically find the eigenvalues of a matrix (i.e. solve the eigenvalue equation) without having to manually do the algebra.

- Multivariable Calculus
  - Scalar and vector functions:
    - Example: $f(x,y,z) = x^2 y + \sin z$ is a scalar function of x, y, z.
    - Example: $\mathbf{F}(x,y,z) = (x^2 y + \sin z, xyz, 4)$ is a vector function of x, y, z. Equivalent statements are:
      - $\mathbf{F}(x,y,z) = (x^2 y + \sin z)\hat{\mathbf{x}} + xyz\hat{\mathbf{y}} + 4\hat{\mathbf{z}}$
      - $F_x = x^2 y + \sin z,\; F_y = xyz,\; F_z = 4$

- Gradient of a scalar function: $\nabla f = \left(\frac{\partial f}{dx}, \frac{\partial f}{dy}, \frac{\partial f}{dz}\right)$, which is a vector function
- Divergence of a vector function: $\nabla \cdot \mathbf{F} = \frac{\partial F_x}{dx} + \frac{\partial F_y}{dy} + \frac{\partial F_z}{dz}$, which is a scalar function
- Curl of a vector function: $\nabla \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$, which is a vector function
- Divergence theorem: $\oint_{\substack{closed \\ surface}} \mathbf{F} \cdot d\mathbf{A} = \int_{\substack{volume\ bounded \\ by\ the\ surface}} \nabla \cdot \mathbf{F}\, dv$
- Stokes' theorem: $\oint_{\substack{closed \\ path}} \mathbf{F} \cdot d\boldsymbol{\ell} = \int_{\substack{surface\ bounded \\ by\ the\ path}} (\nabla \times \mathbf{F}) \cdot\, d\mathbf{A}$
    - Both of those last two theorems are summarized like this: the integral of a function over a closed boundary can be obtained by adding up (integrating) a derivative over the region of space being bounded.

Computational Skills

- Know how to use a program such as Mathematica to do the following:
    - Symbolic integrals
    - Symbolic algebra, if desired
    - Numeric integrals
    - Numeric root finding, including finding the intersection of two functions
    - Matrix inverses
    - Matrix multiplication
    - Matrix eigenvalues
    - Plotting
- Know how to do basic programming in a language of your choice