Vibrational Modes in a Solid

molecules 3 translational d.f
2-3 rotational d.f
? vibrational d.f

Looking ahead

Solid $\Rightarrow$ can use translational symmetry to simplify vibrational problem

- longitudinal vs transverse
- long wavelength (solid like homogeneous medium) $\rightarrow$ but not isotropic vs. short wavelength (must consider microscopic structure)

long wavelength $\downarrow$ rest of this chapter

short wavelength $\downarrow$ Next two chapters

Reminder: matrix multiplication. change a vector into 3 new vector components

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}' = \begin{pmatrix} & b & \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$a_j' = \sum_{k=1}^{3} b_{jk} a_k$$

To turn a matrix into a new matrix components?

$$b_{jk}' = \sum_{l} \sum_{m} C_{jklm} b_{lm}$$

b_{jk} = "rank 2 tensor" $\rightarrow$ 9 components

C_{jklm} = "rank 4 tensor" $\rightarrow$ 81 components

Elastic Strain

From Physics 121

stress = Force/area

strain = $\Delta L/L$

Young's modulus Y = stress/strain

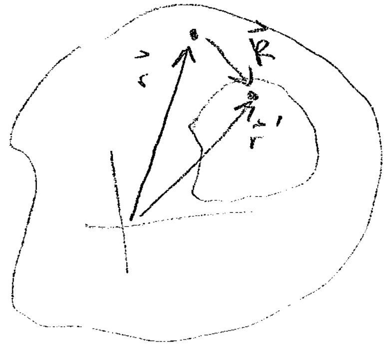
Crystals: difference between eg [100] and [111] directions complicates things!

Goals: • Find generalization of Y, called "stiffness constants" (tensor)

• For ~~[illegible]~~ cubic crystals, get wave eqn for long wavelength waves

→ properties like wave speed (transverse + long.) in [100], [110], [111] (for long) directions in terms of stiffness constants day 10 pg 3

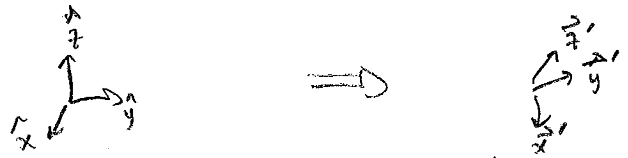
Generalized Strain



$\vec{R}(\vec{r})$ tells how crystal at position $\vec{r}$ gets displaced

$$\vec{R}(\vec{r}) = \underbrace{u\,\hat{x} + v\,\hat{y} + w\,\hat{z}}_{\text{three new functions describing displacement in } x, y, z \text{ directions}}$$

Coordinate axes also transform



(not necessarily unit length)

Define strain in terms of new axes' departure from "expected"

$$e_{xx} = \vec{x}\,' \cdot \vec{x}\,' - 1 = \frac{\partial u}{\partial x}$$
$$e_{yy} = \vec{y}\,' \cdot \vec{y}\,' - 1 = \frac{\partial v}{\partial y}$$
$$e_{zz} = \vec{z}\,' \cdot \vec{z}\,' - 1 = \frac{\partial w}{\partial z}$$
$$e_{yz} = \vec{y}\,' \cdot \vec{z}\,' = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$
$$e_{xz} = \vec{x}\,' \cdot \vec{z}\,' = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$
$$e_{xy} = \vec{x}\,' \cdot \vec{y}\,' = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

derivatives derived in book, for small changes

At least, I think this is what book is doing!

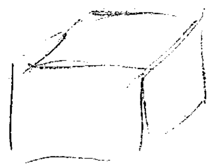
Strain matrix or tensor

Symmetric !

$$\begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{xy} & e_{yy} & e_{yz} \\ e_{xz} & e_{yz} & e_{zz} \end{pmatrix}$$

Shown in book $\dfrac{\Delta V}{V} = e_{xx} + e_{yy} + e_{zz}$

Generalized stress



σ (not used in Kittel)

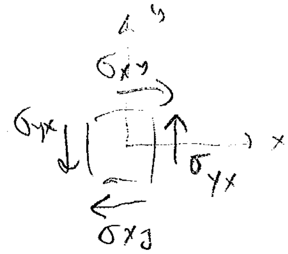
$\sigma_{ij} =$ i j

compressive: $\sigma_{xx} =$ force in x direction per area applied to area, normal in x-dir

shear: $\sigma_{xy} =$ y

Note: $\sigma_{xy} = \sigma_{yx}$

otherwise torque / ang. accel



σ_{ij}: symmetric matrix, 6 independent components

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy} & \sigma_{yz} \\ '' & & \sigma_{zz} \end{pmatrix}$$

write as 6 component vector

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix}$$

same thing w/ strain matrix: 6 component vector.

Relating stress to strain

$$\text{or} \begin{pmatrix} e_1 \\ \vdots \\ e_6 \end{pmatrix} \quad \begin{pmatrix} e_{xx} \\ \vdots \\ e_{xy} \end{pmatrix} = \begin{pmatrix} S \\ \text{"compliance tensor"} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \vdots \\ \sigma_{xy} \end{pmatrix} \quad \text{or} \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_6 \end{pmatrix}$$

to translate back, $S_{23} = S_{(yy)(zz)}$

pos. 2 pos. 3

but only 36 components (potentially) instead of 81

S describes how easily substance is deformed

inverse matrix

C = "stiffness tensor" (they got the letters backwards! :-))

$$\begin{pmatrix} \sigma \end{pmatrix} = \begin{pmatrix} C \end{pmatrix} \begin{pmatrix} e \end{pmatrix}$$