cubic crystal — use symmetry to figure out which elements are 0, and which are equal to others

Consider $C_{14} = C_{xx\,yz}$

$$\begin{pmatrix} \sigma_{xx} \\ \vdots \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} & & C_{14} & & \\ & & & & \\ & & & & \end{pmatrix} \begin{pmatrix} e_{xx} \\ \vdots \\ e_{xy} \end{pmatrix}$$

$$\sigma_{xx} = \ldots + C_{14}\, \underbrace{e_{yz}}_{\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}} + \ldots$$

Reflection symmetry: $z \to -z$ $\quad e_{yz} \to -e_{yz}$ because $\frac{\partial v}{\partial z} \to -\frac{\partial v}{\partial z}$ and $\frac{\partial w}{\partial y} \to -\frac{\partial w}{\partial y}$

perform operation:

$$\sigma_{xx} = \ldots + C_{14}(-e_{yz}) + \ldots$$

can only be equal if $\boxed{C_{14} = 0}$

Similarly 3-fold rotation symmetry about [111] axis $\quad x \to y,\ y \to z,\ z \to x$

forces $\underset{xxxx}{C_{11}} = \underset{yyyy}{C_{22}} = \underset{zzzz}{C_{33}}$ (call it C_{11})

and $C_{44} = C_{55} = C_{66}$

$C_{yzyz} \quad C_{xzxz} \quad C_{xyxy}$

by rotation C_{zxzx} which equals this

Result:

$$\boxed{C = \begin{pmatrix} C_{11} & C_{12} & C_{12} & & & \\ C_{12} & C_{11} & C_{12} & & 0 & \\ C_{12} & C_{12} & C_{11} & & & \\ & & & C_{44} & & 0 \\ & 0 & & & C_{44} & \\ & & & 0 & & C_{44} \end{pmatrix}}$$

only 3 independent components! (down from 81!)

see tables 11 + 12, pg 84 for values of C_{11}, C_{12}, C_{44}

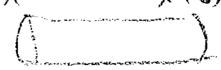
Newton's 2nd Law: $\Sigma F = ma \rightarrow \frac{F}{Vol} = \frac{m}{Vol} \cdot a$

$$\frac{F}{area} \cdot \frac{1}{\Delta x} = \rho \cdot a$$

$\hookrightarrow$ stress = C · strain

1D case

x $\quad\quad$ $x+\Delta x$



$\left.\frac{\partial u}{\partial x}\right|_{x}$ $\quad\quad$ $\left.\frac{\partial u}{\partial x}\right|_{x+\Delta x}$

net strain = $\left.\frac{\partial u}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial u}{\partial x}\right|_{x}$

$$\frac{\Delta\, strain}{\Delta x} \times C = \rho \times accel.$$

$$\underbrace{\frac{\left[\left.\frac{\partial u}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial u}{\partial x}\right|_{x}\right]}{\Delta x}}_{\frac{\partial^2 u}{\partial x^2}} \times C = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{C}{\rho} \frac{\partial^2 u}{\partial x^2}$$

wave eqn! $\quad V = \sqrt{\frac{C}{\rho}}$

proof: plug in $u = u_0 e^{i(kx - \omega t)}$ trial soln

solve for $V = \frac{\omega}{k}$

3D: more possible forces

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}$$

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & & 0 & \\ C_{12} & C_{11} & C_{12} & & & \\ C_{12} & C_{12} & C_{11} & & & \\ & & & C_{44} & & 0 \\ & 0 & & & C_{44} & \\ & & & 0 & & C_{44} \end{pmatrix} \begin{pmatrix} e_{xx} \\ \vdots \\ \vdots \end{pmatrix}$$

Look at big matrix & get $\sigma_{xx} = C_{11} e_{xx} + C_{12}(e_{yy} + e_{zz})$

$$\sigma_{xy} = C_{44} e_{xy}$$

$$\sigma_{xz} = C_{44} e_{xz}$$

Plug in $e_{xx} = \frac{\partial u}{\partial x}$, $e_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$, etc

Result

$$\boxed{\rho \frac{\partial^2 u}{\partial t^2} = C_{11} \frac{\partial^2 u}{\partial x^2} + C_{44}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + (C_{12}+C_{44})\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right)}$$

eqn 3.57a pg 80

Similar eqns for $\frac{\partial^2 v}{\partial t^2}$, $\frac{\partial^2 w}{\partial t^2}$

[typo in eqn 51 on pg 80]

This gives us everything!

100 long.: $u = u_0 e^{i(kx - \omega t)}$ $\quad V=0 \quad W=0$

→ wave vector ~~motion~~ in x direction only

→ oscillation in x dir only

$$\frac{\partial^2 u}{\partial t^2} \to -\omega^2 u$$

$$\frac{\partial^2 u}{\partial x^2} \to -k^2 u$$

$$\frac{\partial}{\partial y}, \frac{\partial}{\partial z} \to 0$$

Eqn becomes:

$$\rho(-\omega^2) = C_{11}(-k^2) + 0$$

$$\boxed{\frac{\omega}{k} = \sqrt{\frac{C_{11}}{\rho}}}$$

100 transv

(same as 010 transv) ~~(same as 100 but ... eqn for ...)~~

$u = U_0 e^{i(ky - \omega t)}$

→ transverse!

Eqn becomes

$$\rho(-\omega^2) = 0 + C_{44}(-k^2 + 0) + 0$$

$$\boxed{\frac{\omega}{k} = \sqrt{\frac{C_{44}}{\rho}}}$$

other transverse: same speed

day 11 pg 4

Math review:

$\lambda x_1 = 3x_1 + 4x_2$

$\lambda x_2 = 5x_1 + 6x_2$

what λ's are allowed?

answer: $\lambda = -.21699$

or $\lambda = 9.21699$

[110], longitudinal

$$u = u_0 e^{i\left(k\left(\frac{x+y}{\sqrt{2}}\right) - \nu t\right)}$$

traveling in 110

$$v = v_0 e^{i(\text{same})}$$

u-eqn: $\rho(-\omega^2)u = C_{11}\left(-\frac{k^2}{2}\right)u + C_{44}\left(-\frac{k^2}{2} + 0\right)u + (C_{12}+C_{44})\left(-\frac{k^2}{2}v + 0\right)$

$$\rho\omega^2 u = \tfrac{1}{2}k^2(C_{11}+C_{44})u + \tfrac{1}{2}k^2(C_{12}+C_{44})v$$

day 12 pg 1

v-eqn $\rho\omega^2 v = \tfrac{1}{2}k^2(C_{11}+C_{44})v + \tfrac{1}{2}k^2(C_{12}+C_{44})u$

Combine:
$$\begin{pmatrix}\rho\omega^2 & 0\\ 0 & \rho\omega^2\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} = \tfrac{1}{2}k^2\begin{pmatrix}C_{11}+C_{44} & C_{12}+C_{44}\\ C_{12}+C_{44} & C_{11}+C_{44}\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}$$

$$\begin{pmatrix}A-\rho\omega^2 & B\\ B & A-\rho\omega^2\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} = 0$$

only solns if matrix is not invertible

$$\det\begin{pmatrix}A-\rho\omega^2 & B\\ B & A-\rho\omega^2\end{pmatrix} = 0$$

$$(A-\rho\omega^2)^2 - B^2 = 0$$

$$A - \rho\omega^2 = \pm B$$

$$\tfrac{1}{2}k^2(C_{11}+C_{44}) - \rho\omega^2 = \pm(C_{12}+C_{44})\tfrac{1}{2}k^2$$

$$2\rho\omega^2 = k^2(C_{11}+C_{44}) \mp (C_{12}+C_{44})$$

$$\frac{\omega^2}{k^2} = \boxed{\frac{1}{2\rho}(C_{11}+C_{12}+2C_{44})} \to \text{long.}$$

or $\boxed{\frac{1}{2\rho}(C_{11}-C_{12})}$ → transverse!

"Effective elastic constants"

why both long + transverse?

We didn't specify relationship between u and v.

How to tell? plug back into u-eqn + v eqn

if $u = v$ then [110], long. displacement

$u = -v$ [$1\bar{1}0$], transv displacement,

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