~~day 12 pg 1~~

day 11 pg 4

[110], longitudinal

$$u = u_0 \, e^{i\left(k\left(\frac{x+y}{\sqrt{2}}\right) - \omega t\right)}$$

↑ traveling in 110

$$v = v_0 \, e^{i(\text{same})}$$

Math review:
$\lambda x_1 = 3x_1 + 4x_2$
$\lambda x_2 = 5x_1 + 6x_2$
What λ's are allowed?
answer: $\lambda = -.21699$
or $\lambda = 9.21699$

u-eqn: $\rho(-\omega^2)u = C_{11}\left(-\frac{k^2}{2}\right)u + C_{44}\left(-\frac{k^2}{2} + 0\right)u + (C_{12}+C_{44})\left(-\frac{k^2}{2} v + 0\right)$

$$\rho\omega^2 u = \tfrac{1}{2}k^2(C_{11}+C_{44})u + \tfrac{1}{2}k^2(C_{12}+C_{44})v$$

day 12 pg 1

v-eqn $\quad \rho\omega^2 v = \tfrac{1}{2}k^2(C_{11}+C_{44})v + \tfrac{1}{2}k^2(C_{12}+C_{44})u$

Combine:

$$\begin{pmatrix} \rho\omega^2 & 0 \\ 0 & \rho\omega^2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \tfrac{1}{2}k^2 \begin{pmatrix} C_{11}+C_{44} & C_{12}+C_{44} \\ C_{12}+C_{44} & C_{11}+C_{44} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{pmatrix} A-\rho\omega^2 & B \\ B & A-\rho\omega^2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 0$$

only solns if matrix is not invertible

$$\det\begin{pmatrix} A-\rho\omega^2 & B \\ B & A-\rho\omega^2 \end{pmatrix} = 0$$

$$(A-\rho\omega^2)^2 - B^2 = 0$$

$$A - \rho\omega^2 = \pm B$$

$$\tfrac{1}{2}k^2(C_{11}+C_{44}) - \rho\omega^2 = \pm(C_{12}+C_{44})\tfrac{1}{2}k^2$$

$$2\rho\omega^2 = k^2\left(C_{11}+C_{44}\right) \pm (C_{12}+C_{44})$$

$$\frac{\omega^2}{k^2} = \boxed{\frac{1}{2\rho}\left(C_{11}+C_{12}+2C_{44}\right)} \rightarrow \text{long.}$$

or $\boxed{\frac{1}{2\rho}\left(C_{11}-C_{12}\right)}$ → transverse!

"Effective elastic constants"

why both long + transverse?

We didn't specify relationship between u and v.

How to tell? plug back into u-eqn + v eqn

if $u = v$ then [110], long. displacement

$u = -v$ $\quad [\bar{1}10]$, transv. displacement, Shortcut way (recommended for 111 HW problem)

Enforce $u = v$ from the start

[110] longitudinal

can do it with single eqn:

$$\rho(-\omega^2)u = C_{11}\left(-\frac{k^2}{2}\right)u + C_{44}\left(-\frac{k^2}{2}+0\right)u + (C_{12}+C_{44})\left(-\frac{k^2}{2}u + 0\right)$$

↑ instead of v!

$$\frac{\omega^2}{k^2} = \frac{1}{\rho}\left[\frac{C_{11}}{2} + \frac{C_{44}}{2} + \frac{C_{12}}{2} + \frac{C_{44}}{2}\right]$$

$$v = \sqrt{\frac{\omega}{k}} = \sqrt{\frac{1}{2\rho}\left(C_{11} + C_{12} + 2C_{44}\right)}$$ ✓

[110] transverse. Final term is $+\frac{k^2}{2}u$

↙ oscill in $[1\bar{1}0]$ direction

$$\frac{\omega^2}{k^2} = \frac{1}{\rho}\left\{\frac{C_{11}}{2} + \frac{C_{44}}{2} - \frac{C_{12}}{2} - \frac{C_{44}}{2}\right)$$

$$v = \sqrt{\frac{\omega}{k}} = \sqrt{\frac{1}{2\rho}\left(C_{11} - C_{12}\right)}$$ ✓

110 other transverse (z-direction) [001] oscillation

$W = W_0 e^{i(k \frac{x+y}{\sqrt{2}} - \omega t)}$

$u, v = 0$

really 3x3 matrix $\left(\begin{array}{c|c} x\,y & 0 \\ \hline 0 & z \end{array}\right)$

→ turns out

$$\frac{\omega}{k} = \sqrt{\frac{C_{44}}{\rho}}$$

---

111 · for HW

long: $C_{eff} = \frac{1}{3}(C_{11} + 2C_{12} + 4C_{44})$

trans 1: $C_{eff} = \frac{1}{3}(C_{11} - C_{12} + C_{44})$

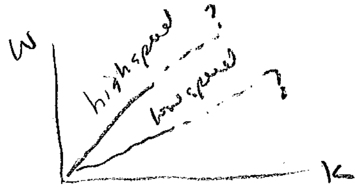
trans 2: same thing!

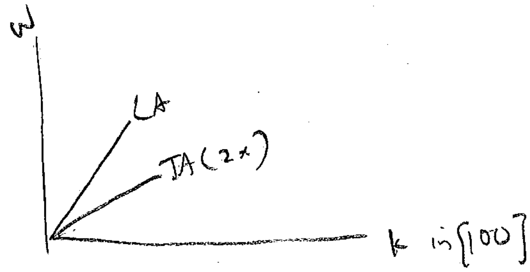
Fig 20 pg 83

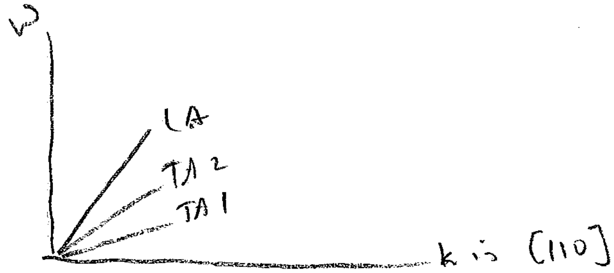
Dispersion relation

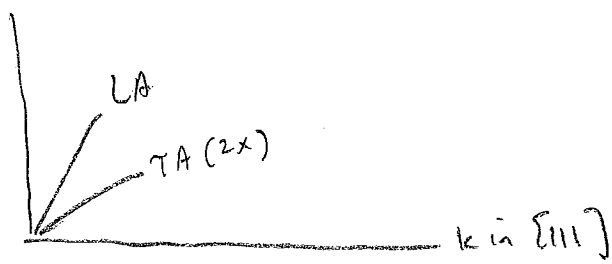
$v = \frac{\omega}{k}$ → only true for long λ (small k)

"acoustic waves"









With computers calc likely do arbitrary direction in BZ $\omega(\vec{k})$

Done w/ Ch 3

skipped "Elastic Energy Density" pg 77-78

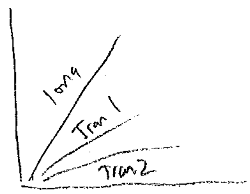
and "Bulk Modulus & Compressibility" pg 80

Chapter 4: Phonons

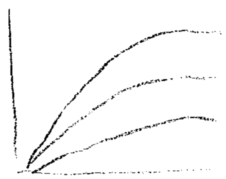
So far we've focused on long wavelength oscillations (small k)

There w vs k is straight line, and slope is speed of elastic wave (HW problem 3.6)

Figure 11 pg 101



what about larger k?

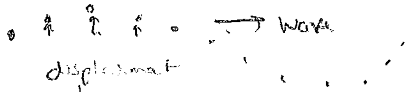


↑ why is there a kmax?

Now we'll answer some of these questions

Basic answer: Leaving the continuum limit!

Consider $\lambda = 10a \Rightarrow k = \frac{2\pi}{10a} = \frac{\pi}{5a}$



But what if $\lambda = a$? $k = 2\pi/a$

is there a wave?

$\lambda = a/2$ $k = 4\pi/a$

That's why there's a kmax! → for $\lambda < a$ doesn't make sense

Fig 5 pg 93 — solid curve contains no more info than dashed curve

only k in 1st BZ are needed!