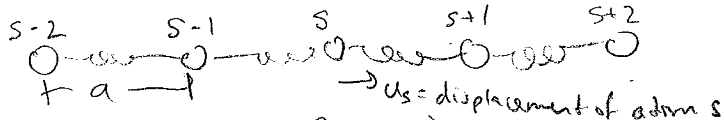
day 13 pg 1

phase vs group velocity

Fresh model: For now all identical items

"Harmonic Approximation"



(or could be pieces of atoms)

Newton 2: $\Sigma F_s = m a_s$

$$\underbrace{\underset{\text{spring constant}}{C}(u_{s+1}-u_s)}_{\text{force from right-hand spring}} - \underbrace{C(u_s - u_{s-1})}_{\text{left hand}} = m\frac{d^2u_s}{dt^2}$$

$$m\frac{d^2u_s}{dt^2} = C\left[u_{s+1} - 2u_s + u_{s-1}\right]$$

Guess $u \sim e^{i(kx-\omega t)}$

$$\text{at } x = \underbrace{s}_{\text{integer}}a \quad \text{(discrete variable)}$$

$$u_{s+1} = e^{ika}u_s$$

Plug in to eqn:

$$m(-\omega^2)\cancel{u_s} = c\left[e^{ika} - 2 + e^{-ika}\right]\cancel{u_s}$$

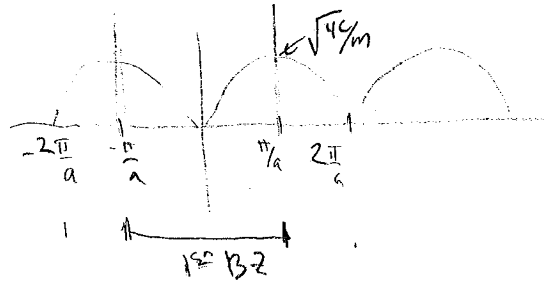
$$e^{ika} + e^{-ika} = 2\cos ka$$

$$m(\omega^2) = 2C\left[1 - \cos ka\right]$$

$$\boxed{\omega = \left[\frac{2c}{m}(1-\cos ka)\right]^{1/2}}$$

dispersion relation!

$$\boxed{\omega = \sqrt{\frac{4c}{m}}\left|\sin\frac{ka}{2}\right|}$$ because $\sin^2\frac{x}{2} = \frac{1}{2} - \frac{1}{2}\cos x$



TOPS 35500

day 13 pg 2

initial slope?

$$\frac{d\omega}{dk} = \sqrt{\frac{4c}{m}} \cos\frac{ka}{2} \left(\frac{a}{2}\right) \Bigg|_{\text{small } k}$$

$$v = \frac{a}{\cancel{2}}\sqrt{\frac{\cancel{4}c}{m}} = a\sqrt{\frac{c}{m}}$$

informative? Starts out linear, at least.

Periodicity? repeats over $\frac{ka}{2} = n\pi$ ($|\sin x|$ has period π, not 2π)

$$k = \frac{2\pi}{a} n$$

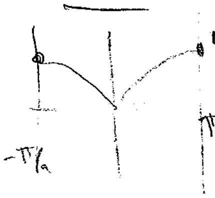
So $k' = k + \frac{2\pi n}{a}$ will have same value as k

What's special about $\frac{2\pi}{a}, \frac{4\pi}{a}, \frac{6\pi}{a}$, etc ?

Reciprocal Lattice Vectors!

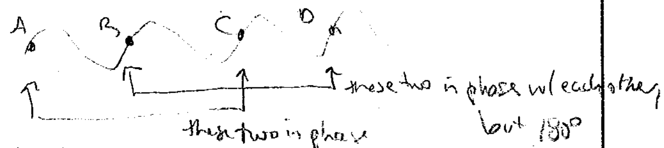
Summary: by subtracting an appropriate RLV from k, we always obtain an equivalent wavevector in 1^{st} BZ

Boundaries



flat slope $\rightarrow v = 0$
Does that make sense?

Yes: $k = \pm\frac{\pi}{a} \rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{a} \longrightarrow \lambda = 2a$



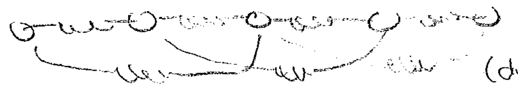
↑ ↓ ↑ ↓

later time

↓ ↑ ↓ ↑

This is a standing wave! Of course no velocity.

Possible extension forces between next nearest neighbors



(diff spring constants)

or between 3^{rd} nearest neighbors ... or 4^{th}, etc.

result: C_p = spring constant between p^{th} nearest neighbors

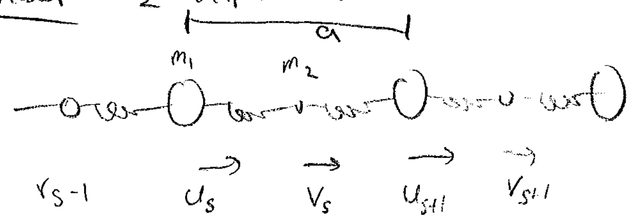
$$\omega = \left[\frac{2}{M} \sum_{p=1}^{\infty} C_p (1 - \cos pka) \right]^{1/2}$$

book shows inverse relationship is

$$C_p = -\frac{Ma}{2\pi} \int_{-\pi/a}^{\pi/a} \omega^2 \cos pka \, dk$$

day 13 pg 4

Next model: 2 different atoms



$$m_1 \frac{d^2 u_s}{dt^2} = C\left[(v_s - u_s) - (u_s - v_{s-1})\right]$$

$\frac{d^2 u_s}{dt^2} \to -\omega^2 u_s$ $\qquad v_{s-1} \to v_s e^{-ika}$

$$\boxed{+\omega^2 m_1 u_s = +2C\, u_s - C\left(1+e^{-ika}\right) v_s}$$

Similarly $\boxed{+\omega^2 m_2 v_s = -C\left(1+e^{+ika}\right) u_s + 2C v_s}$ coupled equations!

typo in Eqn 21 pg 98!

Solve w/ Matrix:
$$0 = \begin{pmatrix} 2C - \omega^2 m_1 & -C\left(1+e^{-ika}\right) \\ -C\left(1+e^{+ika}\right) & 2C - \omega^2 m_2 \end{pmatrix} \begin{pmatrix} u_s \\ v_s \end{pmatrix}$$

$\to$ det $= 0$ for non trivial soln

day 14 pg 1

$$\left(2C - \omega^2 m_1\right)\left(2C - \omega^2 m_2\right) - C^2 \underbrace{\left(1+e^{+ika}\right)\left(1+e^{-ika}\right)}_{1 + e^{ika} + e^{-ika} + 1} = 0$$

$= 2 + 2\cos ka$

$= 2C^2(1+\cos ka)$

$$m_1 m_2 \omega^4 - 2C(m_1+m_2)\omega^2 + C^2\left(4 - 2(1+\cos ka)\right) = 0$$

$4 - 2 - 2\cos ka$

$2C^2(1 - \cos ka) = 0$

quadratic eqn in ω^2

$$\omega^2 = \frac{+\cancel{2}C(m_1+m_2) \pm \sqrt{\cancel{4}C^2(m_1+m_2)^2 - \cancel{4}(m_1 m_2)(2C^2(1-\cos ka))}}{\cancel{2} m_1 m_2}$$

$\omega = \sqrt{}$ $\to$ All that

plotted w/ mathematica for

$m_1 = 3$, $m_2 = 1$, $C = 1$, $a = 1$

$\boxed{\text{Hand out to Class}}$

TOPS. 35500