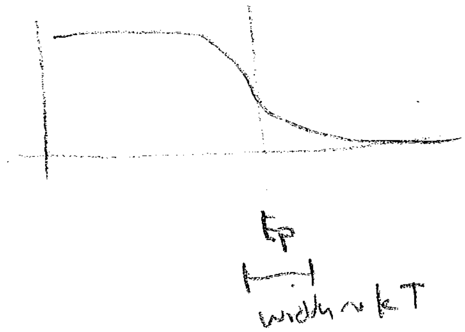
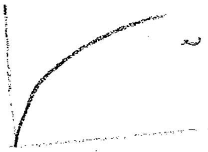
Qualitative calculation of C_V

At low Ts 1) $f(E)$ looks like

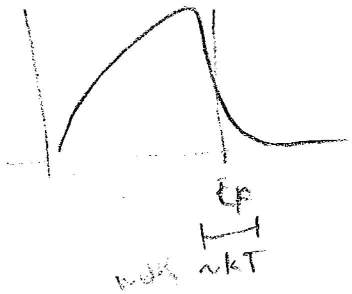


2) $D(E) =$



$\sim E^{1/2}$

Multiply together



how many states there? (that are involved in "action")

$$= N \times \frac{kT}{E_F}$$

if each one gains kT,

then

$$U = N \cdot \frac{k^2T^2}{E_F}$$

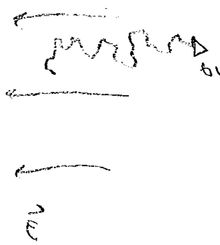
$$\boxed{C_V = 2\,\frac{Nk^2T}{E_F}}$$

compare to $\underbrace{\frac{\pi^2}{2}}_{\text{constant a little off}} \underbrace{N\frac{k^2T}{E_F}}_{\text{right on!}}$

(cont. in Ch 9)

Electrical Conductivity

Classical scattering



$$a = \frac{\Delta V}{\Delta t} = \frac{V_d}{\tau}$$ ← because y + z directions will average to zero, and x direction will average to V_d

also $\sum \vec{F} = m\vec{a}$

$$a = \frac{-e\vec{E}}{m}$$

$$\underline{\underline{V_d = \frac{-eE}{m}\tau}}$$

Review: did this last chapter

$J = \frac{\text{current}}{\text{area}} = \frac{\text{charge}}{\text{area}\cdot\text{time}} = n\,(-e)\,V_d$

"charge flux", I guess; n: $\frac{\text{electrons}}{\text{area}\cdot\text{length}}$; $(-e)$: $\frac{\text{charge}}{\text{electron}}$; V_d: $\frac{\text{length}}{\text{time}}$

$$\boxed{J = \underbrace{\frac{ne^2\tau}{m}}_{\sigma}\cdot E}$$

values of σ in Table 3 pg 149

Ohm's Law! $J = \sigma E$

$$\frac{I}{A} = \sigma\,(V/\ell) \rightarrow V = I \times \text{constant}$$

$$R = \frac{1}{\sigma}\frac{\ell}{A}$$

$$R = \rho\frac{\ell}{A}$$

familiar?

Discrete k values

$$\sum F = m\frac{d\vec{v}}{dt} \rightarrow = \frac{d\vec{p}}{dt} = \hbar\frac{d\vec{k}}{dt}$$

$$= -e\vec{E} = \hbar\frac{d\vec{k}}{dt}$$

$$\vec{k} = \vec{k_0} - \frac{e\vec{E}}{\hbar}t$$

Fermi sphere — no net momentum

now there is a net momentum

scatterings: k_x doesn't get arbitrarily big but reaches some steady-state value

Things match with picture above:

$$k = k_0 - \underbrace{\frac{eE}{\hbar}\tau}$$ ← constant offset (breaks down for large E)

Electrical conduct. $\sigma = \frac{n e^2 \tau}{m}$, values in Table 3 on pg 149

Recall last chapter (day 20)

thermal cond. of phonons $k_{th} = \frac{1}{3} C v \ell$ → "heat cap / volume"

values in Table 1 pg 116

→ model: scattering / mean free path

What about thermal cond. of electrons?

Some results

$$k_{el} = \frac{1}{3} C_{\frac{electrons}{Vol.}} v_{Fermi} \ell_{electrons} \rightarrow v_F \cdot \tau$$

since only electrons on Fermi surface can change energy

$\frac{1}{2} m v_F^2 = E_F$

from a couple of days ago, (+ handout)

$$C_V = \frac{\pi^2}{3} D(E_F) k_B^2 T$$

$$= \frac{\pi^2}{3} \left[\frac{3}{2} \frac{N}{E_F}\right] k_B^2 T$$

Combine

$= v_F^2 \tau$

$= \frac{2E_F}{m} \tau$

$$k_{el} = \frac{1}{3} \left(\frac{\pi^2}{2} \frac{N}{\cancel{E_F}} k^2 T \right] \frac{1}{V} \left[\frac{2 \cancel{E_F}}{m} \tau \right]$$

$$\boxed{k_{el} = \frac{\pi^2}{3} n \frac{k^2 T}{m} \tau}$$

ratio: $$\frac{k_{el}}{\sigma_{el}} = \frac{\frac{\pi^2}{3} \cancel{n} \frac{k^2 T}{\cancel{m}} \cancel{\tau}}{\cancel{n} \frac{e^2 \cancel{\tau}}{\cancel{m}}} = \boxed{\frac{\pi^2}{3} \frac{k_B^2}{e^2} T}$$

Wiedemann-Franz Law

$\frac{k_{thermal}}{\sigma} \sim T$

"Lorentz number" proportionality constant.

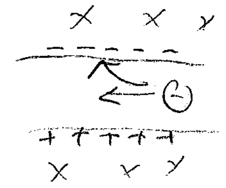
No n! No m!

Independent of material!

$= 2.45 \cdot 10^{-8} \frac{W \Omega}{K^2}$

(mostly) see Table 5 pg 157, for experimental values all very close to this number.

Last item from chapter: magnetic fields



$$\vec F = q\vec E + q\vec v \times \vec B \quad \text{(Phys 220)}$$

$$\vec F = -e\left(\vec E + \vec v \times \vec B\right)$$

also assume a damping term $\sim \vec v$ (like air resistance)

$$\Sigma F = ma \longrightarrow m\frac{d\vec v}{dt} = -e\left(\vec E + \vec v \times \vec B\right) - \frac{m\vec v}{\tau}$$

← characteristic damping time

Steady state: $\frac{d}{dt} \to 0$

$$\frac{m\vec v}{\tau} = -e\left(\vec E + \vec v \times \vec B\right)$$

Typically $\vec E \perp \vec B$ in "Hall Effect" experiments

$\vec E$ in x-y plane; $\vec B$ in $\hat z$

v also in x-y plane

$$\frac{m v_x}{\tau} = -e\left(E_x + v_y \underset{\text{just "B"}}{B_z}\right) \longrightarrow v_x = -\frac{e\tau}{m}E_x - \left(\frac{eB}{m}\right)\tau v_y$$

units of frequency: ω_c "cyclotron"

and

$$\frac{m v_y}{\tau} = -e\left(E_y - v_x B_z\right) \longrightarrow v_y = -\frac{e\tau}{m}E_y + \left(\frac{eB}{m}\right)\tau v_x$$

day 24 pg 1

Now, force $v_y = 0$ for current flow

$$v_x = -\frac{e\tau}{m}E_x \quad ; \quad 0 = -\frac{e\tau}{m}E_y + \left(\frac{eB}{m}\right)\tau v_x$$

$$0 = -\cancel{\frac{e\tau}{m}}E_y + \left(\frac{\cancel{e}B}{\cancel{m}}\right)\cancel{\tau}\left(-\frac{e\tau}{m}E_x\right)$$

$$\boxed{E_y = -\frac{e\tau}{m}B\,E_x}$$

Transverse field! leads to "Hall voltage", what you actually measure

negative number because $e = +1.6 \times 10^{-19}$ C