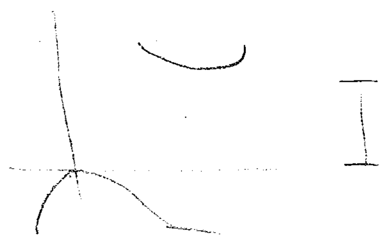
Ch. 8
Effective Mass



realistic band gap opens up due to crystal potential
- near zone edges, it's quadratic.

 ← put electron at bottom of slope, how does it behave?

Free electron $E = \frac{\hbar^2 k^2}{2m} \longrightarrow \frac{dE}{dk} = \frac{\hbar^2}{m} k$

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$$

Electron in crystal potential $\boxed{\frac{1}{m_{eff}} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}}$

Notation: $m_{eff} = m^*$

Rigorously from
$\vec{p} = \hbar \vec{k}$
$\vec{F} = \hbar \frac{d\vec{k}}{dt}$
+ wave group velocity

 heavy mass

 light mass

Semiconductors: $m_{eff} \simeq .01 - .1\, m_{electron}$ (near band gap)

- Newton's 2nd law not violated for crystal as a whole
- It means when electron has force on it by eg. applied E or B field, it gets accelerated more than you would expect because of forces also applied to it by the ions (nuclei) that are causing the overall periodic potential.

tensor

If curvature is different in different directions, then need tensor

$$\left(\frac{1}{m}\right)_{xx} = \frac{1}{\hbar^2}\frac{d^2E}{dk_x^2}$$

$$\left(\frac{1}{m}\right)_{xy} = \frac{1}{\hbar^2}\frac{d^2E}{dk_x dk_y}$$

etc.

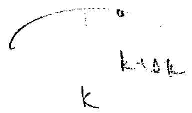
Then N2 is $F = m\frac{dv}{dt} \longrightarrow \frac{dv}{dt} = \frac{1}{m}F \longrightarrow \frac{dv_x}{dt} = \underset{\uparrow\, Y}{\left(\frac{1}{m}\right)_{xx}} F_x + \underset{\uparrow\, Y}{\left(\frac{1}{m}\right)_{xy}} F_y + \underset{\uparrow\, Y}{\left(\frac{1}{m}\right)_{xz}} F_z$

$\frac{dv_y}{dt}$

etc

m_{eff} can even be negative!

what it means = going from k to $k+\delta k$



the lattice causes a bigger $\frac{d\vec{p}}{dt}$ than the applied force does

Correct old eqns

$$\sigma = \frac{ne^2\tau}{m} \longrightarrow \sigma = \frac{ne^2\tau}{m^*}$$ small m → better conductor

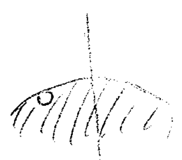
$$\omega_c = \frac{eB}{mc} \longrightarrow \omega_c = \frac{eB}{m^* c}$$ one way to measure m^* is cyclotron resonance

$$D(E) = \frac{V}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{E}$$

$$E_F = \left(\frac{3\pi^2 N}{V}\right)^{2/3}\left(\frac{\hbar^2}{2m^*}\right)$$

etc.

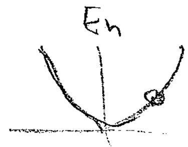
Holes — like auditorium w/ one empty seat. (also like bubbles.)



electron picture

m_e = negative

⟹ hole picture



$$E_h = \frac{\hbar^2 k_h^2}{2m_h}$$

$$m_h = -m_e$$

m_h = positive

$$\boxed{\vec{k}_h = -\vec{k}_e}$$

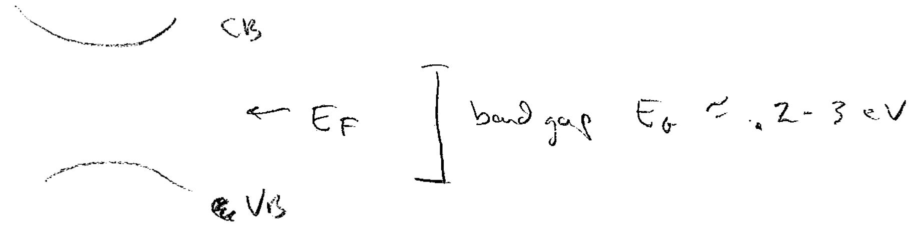
$E_h = -E_e$ → band flipped upside down

Charge = $+|e|$

spin = also reversed

people frequently talk of holes, but don't usually draw valence bands like ∪

Valence + Conduction bands



General Rule (Problem 9.3) – $m^* \sim E_g$, approximately for direct gap materials

[illegible] may or may not assign uses 2nd order perturb. theory, uses bracket notation, would need to translate

Table of E_g: pg 190 Table 1

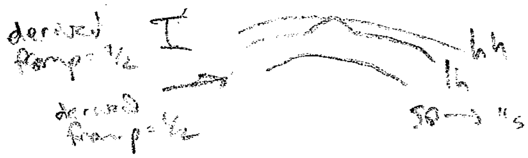
Table of m^*: pg 201 Table 2

handout of Band gap Engineering? Figure? 1 color?

<u>VB details</u>

See Fig 14 pg 203 for Ge

or handout for Si + GaAs?



SO → "split off" by "spin orbit coupling"

← how many electrons naturally up here?

$$f = \frac{1}{e^{(E-\mu)/kT}+1}$$

"tempts of [illegible]" $E-\mu >> kT$

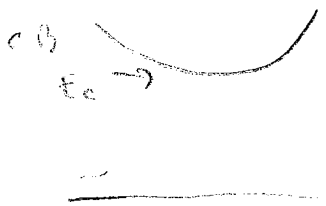
[illegible] energy [illegible]

go already [illegible]?

$$f \approx \frac{1}{e^{(E-\mu)/kT}}$$

$$= e^{-(E-\mu)/kT}$$

Side note, maybe after next [illegible]



← density of states:

just like free electron except

→ m^* to explain different curvature

→ $E - E_c$ to explain vertical offset ←

$$D(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}$$

concentration: $n = \int_{E_c}^{\infty}\left(\frac{D(E)}{V}\right) f(E)\, dE$

$$= \int_{E_c}^{\infty}\left(\frac{1}{2\pi^2}\right)\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}\, e^{-(E-\mu)/kT}\, dE$$

$$= \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} e^{+\mu/kT}\underbrace{\int_{E_c}^{\infty}(E-E_c)^{1/2}e^{-E/kT}\, dE}_{\text{Mathematica: } \frac{\sqrt{\pi}}{2}e^{-E_c/kT}(kT)^{3/2}}$$

$$\boxed{n = 2\left(\frac{m^* kT}{2\pi\hbar^2}\right)^{3/2} e^{-(E_c-\mu)/kT}}$$

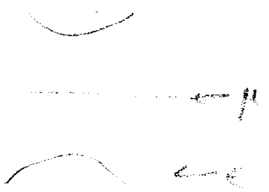
proof of 2's.

$$2 \cdot \frac{2^{-3/2}}{2^{-1/2}} \quad \text{or} \quad \frac{2^{3/2}}{2} \cdot \frac{1}{2} = 2^{3/2}\, 2^{-2} = 2^{-1/2} \checkmark$$

problem: don't know μ.

Do same thing for holes in valence band

$$f_h(E) = 1 - f(E)$$
$$= 1 - \frac{1}{e^{(E-\mu)/kT} + 1} = \frac{e^{(E-\mu)/kT}}{e^{(E-\mu)/kT} + 1}$$



$$\frac{e^{x}}{e^{x}+1} \cdot \frac{e^{-x}}{e^{-x}} = \frac{1}{1+e^{-x}}$$

$$= \frac{1}{e^{(\mu-E)/kT} + 1}$$

$$\approx e^{-(\mu-E)/kT}$$

$\mu - E$ = positive

(E = energy of electrons, really want energy of holes which would have opposite sign)

Same integral [illegible] Typo pg 206

$$p = \int_{-\infty}^{E_v} \frac{D_h(E)}{V} f_h(E)\, dE$$

modified for [illegible]

$$p = 2\left(\frac{m^*_{hole} kT}{2\pi\hbar^2}\right)^{3/2} e^{-(\mu - E_v)/kT}$$ (same integral)

Multiply together

$$np = 4\left(\frac{kT}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} e^{-\frac{1}{kT}(E_c - \cancel{\mu} + \cancel{\mu} - E_v)}$$

$$\boxed{np = 4\left(\frac{kT}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} e^{-E_g/kT}}$$

only assumption: μ far from both CB + VB

could be "intrinsic", could be "doped"

↳ but carefully as 1st condition is still met "nondegenerate"