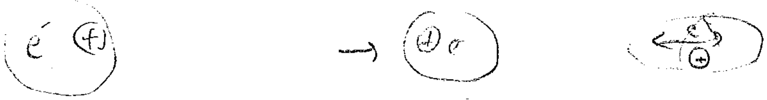
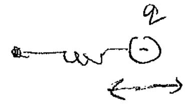
Lorentz model of dielectrics

Apply field | Release | bounces back + forth.



kind of like mass on a spring! (charged mass)



Let's assume there's a resonant freq, ω_0, that describes this.

$$\Sigma F = ma$$

$$m \frac{d^2 \vec{u}}{dt^2} = -m\omega_0^2 \vec{u} + q\vec{E}$$

$\vec{u}$ is displ. from equilibrium

oscillating field from light wave $\vec{E}_0 e^{i(kx-\omega t)}$

Would find an infinite response when $\omega = \omega_0$

... add damping

simplest model, like air resistance $F_{damping} \sim$ velocity, $\frac{d\vec{u}}{dt}$

$$\boxed{\frac{d^2\vec{u}}{dt^2} = -\omega_0^2 \vec{u} + \frac{q}{m}\vec{E}_0 e^{i(kx-\omega t)} - \gamma \frac{d\vec{u}}{dt}}$$

"damped, driven harmonic oscillator"

Guess solution $\vec{u} = \vec{u}_0 e^{i(kx-\omega t)}$

$$-\omega^2 \vec{u}_0 \cancel{e^{i(kx-\omega t)}} = -\omega_0^2 \vec{u}_0 \cancel{e^{i(kx-\omega t)}} + \frac{q}{m}\vec{E}_0 \cancel{e^{i(kx-\omega t)}} - \gamma(-i\omega)\vec{u}_0 \cancel{e^{i(kx-\omega t)}}$$

$$(\omega_0^2 - \omega^2 - i\gamma\omega)\vec{u}_0 = \frac{q}{m}\vec{E}_0$$

$$\vec{u}_0 = \frac{q}{m}\frac{\vec{E}_0}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$\boxed{\vec{u} = \frac{q}{m}\frac{\vec{E}_0}{\omega_0^2 - \omega^2 - i\gamma\omega} e^{i(kx-\omega t)}}$$

dipole moment = $q \times u$

total " " = $N \times q \times u$

$$\vec{P} = \frac{\text{total dipole}}{\text{volume}} = \underbrace{\frac{q^2 n}{m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}}_{\text{this is } \epsilon_0 \chi_e!} \vec{E}_0 \, e^{i(kx - \omega t)}$$

$P = \epsilon_0 \chi_e E$

$$\epsilon_r = 1 + \chi_e$$

$$\boxed{\epsilon_r = 1 + \frac{q^2 n}{\epsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}}$$

units of ω_1^2

call it $\boxed{\omega_p = \sqrt{\frac{q^2 n}{\epsilon_0 m}}}$

"plasma frequency"
we'll discuss why this name in another lecture

If more than one resonant frequency, simply sum! (weighted sum actually)
(eg. because of more than one type of atom)

$$\epsilon_r = 1 + \sum_j \frac{f_j \, \omega_{pj}^2}{\omega_{0j}^2 - \omega^2 - i\omega\gamma_j}$$

weighting factor f_j called "oscillator strength"

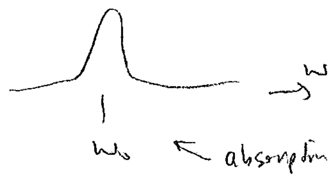
when written with λ instead of ω,
called the "Sellmeier Equation"

wiki: $$n^2 = 1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3}$$

(used for glass)

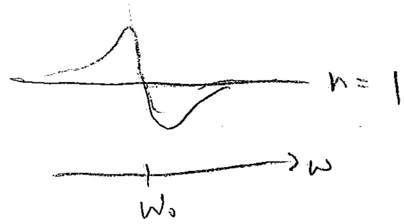
Fig. $n = \sqrt{\epsilon_r}$ → plot real + imag parts of n

Imag. part of $\hat{n}$



← absorption line (for glass, is UV)

Real part of $\hat{n}$

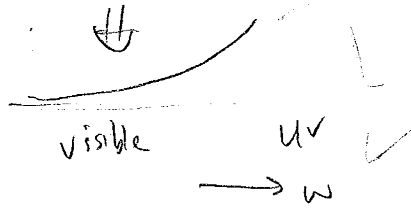


$\Uparrow$

except for this situation, "anomalous dispersion"
index always rises w/ frequency

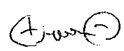
regular "normal dispersion"

$\Downarrow$



<u>Ions</u> — can also act like charged SHOs. Frequencies are much lower

ex. NaCl or GaAs 2 atoms / unit cell, 3D



optical phonons: 1 LO
2 TO

assume degenerate, at ω_T

acoustic 1 LA
2 TA

$$\epsilon = \epsilon_{phonons} + \epsilon_{electrons}$$

$\epsilon_{electrons} \rightarrow \epsilon_\infty$ "high freq. dielectric constant"

hi freq: much higher than vibrational freqs, but below electronic excitation energies

$$\epsilon = \epsilon_\infty + \frac{nq^2}{m\epsilon_0} \frac{1}{\omega_T^2 - \omega^2 - i\omega\gamma}$$

n: # ion pairs / volume

m: reduced mass $\left(\frac{1}{m_1} + \frac{1}{m_2}\right)^{-1}$

$q \le$ valence of ions

change of variables

Note that when $\omega = \omega$

$$\epsilon = \underbrace{\epsilon_\infty + \frac{nQ^2}{m\epsilon_0} \frac{1}{\omega_T^2}}_{\text{"}\epsilon_0\text{"}}$$

always ϵ_r, not ϵ!

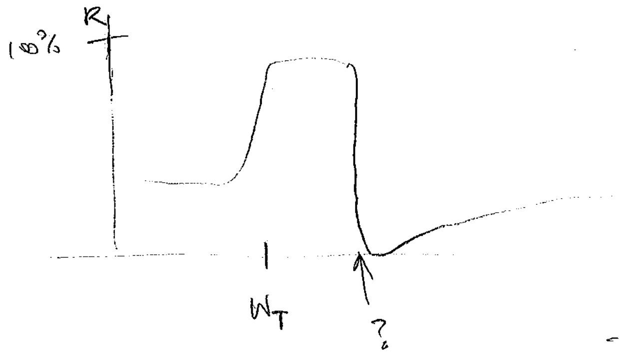
$$\boxed{\epsilon = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2} - i\frac{\omega\gamma}{\omega_T^2}}}$$

→ can calculate $\tilde{n}$

→ can calculate R

typical:
$\epsilon_0 = 15$
$\epsilon_\infty = 12$
$\frac{\gamma}{\omega_T} = .02$ (maybe)

handout Yu + Cardona pg 289, 290 graphs



freq at which $\epsilon = 0$
call it "W_L"

Turns out... W_T: frequency of TO phonons
W_L = " " LO phonons

strange case

regular case wave →

$\ominus \; \oplus \; \ominus \; \oplus$

Electric field

obvious to see why this frequency can be absorbed / gets a response

$\ominus \; \oplus \; \ominus \; \oplus$

wave →

↔ electric field.
can't really do that w/ external lightwave
could happen internally, though

Maxwell Eqn $\nabla \cdot \vec{D} = 0 \rightsquigarrow \nabla \cdot (\epsilon_0 \epsilon_r \vec{E}) = 0$

$\nabla \cdot \vec{E} \longrightarrow \vec{k} \cdot \vec{E}$

but $\vec{k} \cdot \vec{E}$ <u>not</u> zero for this case

so $\boxed{\epsilon_r = 0}$