Lyddane-Sachs-Teller

back to Sellmeier, but w/ $\gamma = 0$

$$\varepsilon = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}}$$

when $\varepsilon = 0$, $\omega = \omega_L$

$$0 = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \underbrace{\frac{\omega_L^2}{\omega_T^2}}_{x}}$$

$$0 = A + \frac{B - A}{1 - x}$$

$$0 = \cancel{A} - Ax + B - \cancel{A}$$

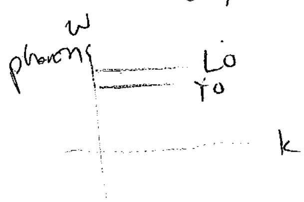
$$x = \frac{B}{A}$$

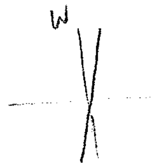
$$\boxed{\left(\frac{\omega_L}{\omega_T}\right)^2 = \frac{\varepsilon_0}{\varepsilon_\infty}}$$ LST

## Polaritons – coupling of em wave w/ something

### phonon polaritons (the only type mentioned in Kittel)

$\oplus \overset{\leftrightarrow}{} \ominus$ optical phonons
ions couple to EM wave.




photons (not quite vertical!)

$$w = vk = \left(\frac{c}{n}\right)k$$

In regions where frequencies are degenerate ($w_T$),
really need degenerate pert. theory (like we did
for band structure when two electron
modes had same frequency).

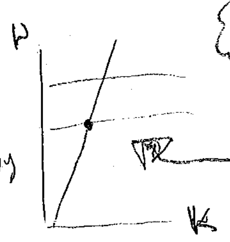
Result → two coupled modes, "upper branch"
+ "lower branch"

At frequencies farther away, looks regular again.

Instead of QM degenerate pert. theory, use this method which gives
essentially correct result.

Kittel Fig 14.11 + 14.12

~~no traveling wave, ε, k is pure imaginary~~




relevant area around $w_{TO} = \left(\frac{c}{n}\right)k$

Force dielectric constants to be the same

Light: $\frac{w}{k} = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon}} \rightarrow \varepsilon = \left(\frac{ck}{w}\right)^2$

TO phonons: $\varepsilon = \varepsilon_\infty + \dfrac{\varepsilon_0 - \varepsilon_\infty}{1 - \dfrac{w^2}{w_T^2}}$ (no damping, for simplicity)

$$\boxed{\frac{c^2k^2}{\omega^2} = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}}}$$

polariton dispersion relation

can in theory solve $\omega$ vs $k$

$\times \omega^2$

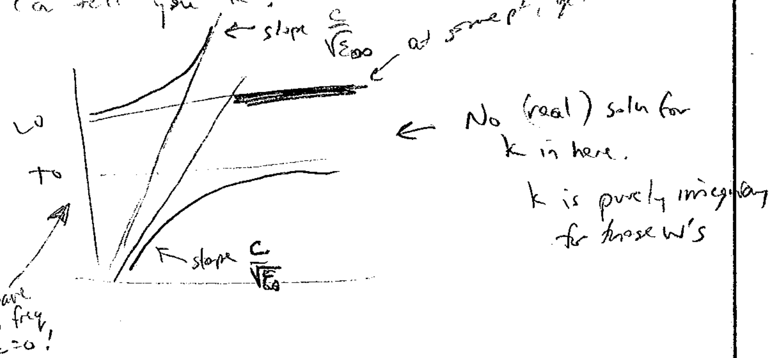
$$c^2k^2 = \varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{1 + \frac{\omega^2}{\omega_T^2}}$$

Hard to solve for $\omega$,

Easier for $k$

$$\boxed{k = \frac{1}{c}\sqrt{\varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{1 + \frac{\omega^2}{\omega_T^2}}}}$$

For a given $\omega$, I can tell you $k$!




Consider ~~the~~ lower branch, small $k$ / small $\omega$

$$k = \frac{1}{c}\sqrt{\varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{1 + \cancel{\text{small}}}}$$

$$\underline{k = \frac{1}{c}\,\omega\sqrt{\varepsilon_0}}$$

Consider upper branch, large $k/\omega$

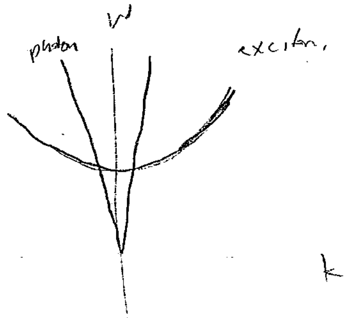
$$k = \frac{1}{c}\sqrt{\varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{\cancel{1} + \frac{\omega^2}{\omega_T^2}}}$$

$$\sqrt{\varepsilon_\infty \omega^2 + \underbrace{\frac{\cancel{\omega^2}(\varepsilon_0 - \varepsilon_\infty)}{\cancel{\omega^2}/\omega_T^2}}_{\text{then this small}}}$$

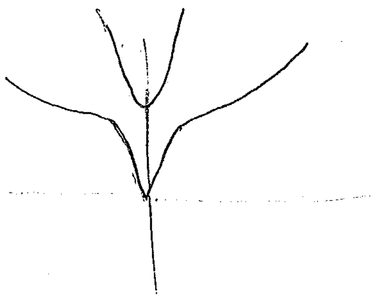
$$\underline{k = \frac{1}{c}\,\omega\sqrt{\varepsilon_\infty}}$$

Exciton polariton

Do excitons interact w/ EM wave? of course!




result



Plasmons
- back to conductors

$$\varepsilon = 1 + \underbrace{\frac{ne^2}{m\epsilon_0}}_{\omega_p^2} \underbrace{\frac{1}{\omega_0^2 - \omega^2}}_{\text{No restoring force now! No resonant freq!}}$$

$$\boxed{\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}} \quad \text{or really} \quad \boxed{\varepsilon = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}} = \varepsilon_\infty\left(1 - \frac{\omega_p^2/\varepsilon_\infty}{\omega^2}\right) = \varepsilon_\infty\left(1 - \frac{\omega_{p\,\text{effective}}^2}{\omega^2}\right)$$

if "positive ion core background has dielectric constant $\varepsilon_\infty$ essentially constant up to frequencies well above $\omega_p$"

Like w/ LO phonon, this has a longitudinal resonance when $\varepsilon = 0$
$\rightarrow \omega = \omega_p$

When $\omega < \omega_p$
$\varepsilon < 0$

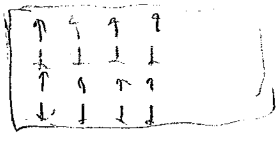
$n = \sqrt{\varepsilon}$ = purely imaginary!

then $R = \left|\frac{1-\tilde{n}}{1+\tilde{n}}\right|^2 = 100\%$ (can show)


Now - mention

At $\omega_p$, $\varepsilon = 0$, $\rightarrow$ longitudinal resonance

Kittel fig 14.4



arrows = displacement of electrons

"plasmon" - a quantized plasma oscillation
come in units of $\hbar\omega_p$