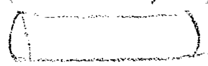
Newton's 2nd Law: $\Sigma F = ma \rightarrow \frac{F}{Vol} = \frac{m}{Vol} a$

$\frac{F}{area} \frac{1}{\partial x} = \rho \cdot a$

↳ stress = c · strain

1D case

x $\quad$ $x+\Delta x$



$\frac{\partial u}{\partial x}\Big|_x \qquad \frac{\partial u}{\partial x}\Big|_{x+\Delta x}$

net strain = $\frac{\partial u}{\partial x}\Big|_{x+\Delta x} - \frac{\partial u}{\partial x}\Big|_x$

$\frac{\text{strain}}{\Delta x} \times C = \rho \times \text{accel.}$

$$\underbrace{\left[\frac{\frac{\partial u}{\partial x}\Big|_{x+\Delta x} - \frac{\partial u}{\partial x}\Big|_x}{\Delta x}\right]}_{\frac{\partial^2 u}{\partial x^2}} \times C = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{C}{\rho} \frac{\partial^2 u}{\partial x^2}$$

wave eqn! $\quad V = \sqrt{\frac{C}{\rho}}$

3D: other ways of getting F_x...

$F_x = \sigma_{xx} A_x + \sigma_{xy} A_y + \sigma_{xz} A_z$

proof: plug in $u = u_0 e^{i(kx - \omega t)}$ trial soln

solve for $V = \frac{\omega}{k}$

Newton 2 Eqn Becomes

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} C_{11} C_{12} C_{12} & & \\ C_{12} C_{11} C_{12} & & \\ C_{12} C_{12} C_{11} & & \\ & C_{44} & 0 \\ & 0 & C_{44} \\ & & C_{44} \end{pmatrix} \begin{pmatrix} e_{xx} \\ \vdots \end{pmatrix}$$

Look at big matrix to get $\sigma_{xx} = C_{11} e_{xx} + C_{12}(e_{yy} + e_{zz})$

$\sigma_{xy} = C_{44} e_{xy}$

$\sigma_{xz} = C_{44} e_{xz}$

Plug in $e_{xx} = \frac{\partial u}{\partial x}$, $e_{xy} = \underbrace{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}_{\text{↳ Kittel's convention}}$, etc

Result

$$\boxed{\rho \frac{\partial^2 u}{\partial t^2} = C_{11}\frac{\partial^2 u}{\partial x^2} + C_{44}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + (C_{12}+C_{44})\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right)}$$

eqn 3.57a pg 80

Similar eqns for $\frac{\partial^2 v}{\partial t^2}$, $\frac{\partial^2 w}{\partial t^2}$

[typo in eqn 51 on pg 80]

This gives us everything!

100 long.: $u = u_0 e^{i(kx - \omega t)}$ $\quad v=0 \quad w=0$

wave ~~vector~~ in x direction only

oscillation in x dir only

$$\frac{\partial^2 u}{\partial t^2} \to -\omega^2 u$$

$$\frac{\partial^2 u}{\partial x^2} \to -k^2 u$$

$$\frac{\partial}{\partial y}, \frac{\partial}{\partial z} \to 0$$

Eqn becomes:

$$\rho(-\omega^2) = C_{11}(-k^2) + 0$$

$$\boxed{\frac{\omega}{k} = \sqrt{\frac{C_{11}}{\rho}}}$$

"Effective elastic constants"

Table in book Figure

100 transv.

(same as 010 transv) ~~(same as 100 but~~ ~~Z comp~~ ~~eqn for w)~~

$u = u_0 e^{i(ky - \omega t)}$

transverse!

Eqn becomes

$$\rho(-\omega^2) = 0 + C_{44}(-k^2 + 0) + 0$$

$$\boxed{\frac{\omega}{k} = \sqrt{\frac{C_{44}}{\rho}}}$$

other transverse: same speed

Started on (110) longitudinal : $k = k \frac{\hat{x}+\hat{y}}{\sqrt{2}}$

$$u = u_x \hat{x} e^{i(k \frac{x+y}{\sqrt{2}} - \omega t)}$$
$$+ u_y \hat{y} e^{i(k \frac{x+y}{\sqrt{2}} - \omega t)}$$

Plug into u_x eqn:

$$\rho(\omega^2) u_x = C_{11}\left(\frac{k^2}{2}\right) u_x + C_{44}\left(-\frac{k^2}{2}\right) u_x + \left((C_{12}+C_{44})\left(\frac{k^2}{2}\right)\right) u_y$$

$$\boxed{\begin{aligned} &= \tfrac{1}{2}k^2(C_{11}+C_{44})u_x + \tfrac{1}{2}k^2(C_{12}+C_{44})u_y \\ u_y \text{ eqn: } \rho\omega^2 u_y &= \tfrac{1}{2}k^2(C_{11}+C_{44})u_y + \tfrac{1}{2}k^2(C_{12}+C_{44})u_x \end{aligned}}$$

Math review: $\lambda x_1 = 3x_1 + 4x_2$

$\lambda x_2 = 5x_1 + 6x_2$

What are possible λ's ?

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 3-\lambda & 4 \\ 5 & 6-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

↳ if this is invertible, get $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

must be noninvertible.

Therefore $\det(M) = 0$

$(3-\lambda)(6-\lambda) - 4 \times 5 = 0$

$\rightarrow \lambda_1 = -.217$

or $\lambda_2 = 9.217$