→ give handout

Finish "hard way" for 110 (Example 4) → For HW... can use Mathematica/Matlab to get eigenvalues + eigenvectors!

Then come back to "easy way" for 110 longitudinal (Example 3)

Example 4 "easy way" for (110) transverse in $\frac{\hat{x}-\hat{y}}{\sqrt{2}}$ direction

$e^{i(k(\frac{x+y}{\sqrt{2}})-\omega t)}$ $u_y = -u_x$

Use Eqn 3.57a

$$\rho^2(\omega^2)u_x = C_{11}\left(\frac{k^2}{2}\right)u_x + C_{44}\left(\frac{k^2}{2}+0\right)u_x + (C_{12}+C_{44})\left(-\frac{k^2}{2}u_x + 0\right)$$

$$\rho\omega^2 = \frac{1}{2}\left[C_{11} + \cancel{C_{44}} - C_{12} - \cancel{C_{44}}\right]k^2$$

$$\boxed{\frac{\omega}{k} = \sqrt{\frac{1}{2\rho}(C_{11}-C_{12})}}$$

✓ matches transverse on handout.

That's not the only transverse direction, though!

$u = (0, 0, u_z)$ is also transverse to (110)

Math → $\frac{\omega}{k} = \sqrt{C_{44}/\rho}$ is result

The three eigenvectors are

(1 1 0) long.

(1 -1 0) transverse 1

(0 0 1) transverse 2

if we hadn't assumed $z=0$ in "the long way", all three would have come out.

Note: 1) All other transverse directions (eg (5, -5, 3)) can be written as linear comb of the two "special" transverse ones.

2) You can't always guess at "special" directions. Sometimes even long. is not an eigenvalue!

How to Solve for wave speed in a given direction - Phys 581

Wave information

1) General exponential dependence is $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$
2) Direction of travel tells you what $\vec{k}$ components are present
3) Direction of oscillation (ie. longitudinal vs transverse, and which specific transverse direction) tells you which $\vec{u}$ components are present.

How to use Eqn 3.57 (a), (b), (c).

1) The Easy Way - If you know the specifics of u_x, u_y, u_z, plug that in from the start.

Example 1 - (100) waves, longitudinal

Given $\vec{k} = (k,0,0)$ and $\vec{u} = u_x \hat{x} e^{i(kx-\omega t)}$

Eqn 3.57 a becomes

$$\rho\omega^2 \cancel{u} = C_{11} k^2 \cancel{u} \rightarrow \boxed{v = \frac{\omega}{k} = \sqrt{\frac{C_{11}}{\rho}}}$$

Example 2 - (100) waves, transverse in the (0,1,0) direction

Given $\vec{k} = (k,0,0)$ and $u = (0,u,0)$

I.e. $\vec{u} = u_y \hat{y} e^{i(kx-\omega t)}$

Eqn 3.57 a becomes

$$\rho\omega^2 \cancel{u} = C_{44} k^2 \cancel{u} \rightarrow \boxed{\frac{\omega}{k} = \sqrt{\frac{C_{44}}{\rho}}}$$

Example 3: (110) waves, longitudinal

Given $\vec{k} = k\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right)$ and $\vec{u} = u\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right) e^{i\left(k\left(\frac{x+y}{\sqrt{2}}\right)-\omega t\right)}$

Eqn 3.57a becomes

$$\rho\omega^2 u = C_{11}\frac{k^2}{2}u + C_{44}\frac{k^2}{2}u + C_{12}+C_{44}\left(\frac{k^2}{2}u\right)$$

$$\rightarrow \boxed{\omega/k = \sqrt{\frac{1}{2\rho}(C_{11}+C_{12}+2C_{44})}}$$

TOPS. 35500

2) The Hard Way – if you don't know all of the specifics, you'll need to solve simultaneous eq'ns in an eigenvalue-type matrix eqn.

Example 4: (110) waves, unknown direction in x-y plane

Given $\vec{k} = k\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right)$, $\vec{u} = \underbrace{(u_x\hat{x}+u_y\hat{y})}_{\text{these unknown. But } u_z=0 \text{ is known.}} e^{i\left(k\left(\frac{x+y}{\sqrt{2}}\right)-\omega t\right)}$

this still known because of k

Eqn 3.57a: $\rho\omega^2 u_x = C_{11}\left(\frac{k^2}{2}\right)u_x + C_{44}\left(\frac{k^2}{2}\right)u_x + (C_{12}+C_{44})\left(\frac{k^2}{2}\right)u_y$

or $\rho\omega^2 u_x = \frac{1}{2}k^2(C_{11}+C_{44})u_x + \frac{1}{2}k^3(C_{12}+C_{44})u_y$

Eqn 3.57b: $\rho\omega^2 u_y = \frac{1}{2}k^2(C_{11}+C_{44})u_y + \frac{1}{2}k^2(C_{12}+C_{44})u_x$

Combine in matrix eqn:

$$\begin{pmatrix}\rho\omega^2 & 0\\ 0 & \rho\omega^2\end{pmatrix}\begin{pmatrix}u_x\\u_y\end{pmatrix} = \frac{1}{2}k^2\begin{pmatrix}C_{11}+C_{44} & C_{12}+C_{44}\\ C_{12}+C_{44} & C_{11}+C_{44}\end{pmatrix}\begin{pmatrix}u_x\\u_y\end{pmatrix}$$

$$\begin{pmatrix}\frac{k^2}{2}(C_{11}+C_{44})-\rho\omega^2 & \frac{k^2}{2}(C_{12}+C_{44})\\ \frac{k^2}{2}(C_{12}+C_{44}) & \frac{k^2}{2}(C_{11}+C_{44})-\rho\omega^2\end{pmatrix}\begin{pmatrix}u_x\\u_y\end{pmatrix} = 0$$

To have non zero solns, $\det(\ \ ) = 0$

$$\left[\frac{k^4}{2}(C_{11}+C_{44})-\rho\omega^2\right]^2 - \left[\frac{k^2}{2}(C_{12}+C_{44})\right]^2 = 0$$

$$\frac{k^2}{2}(C_{11}+C_{44}) - \rho\omega^2 = \pm\frac{k^2}{2}(C_{12}+C_{44})$$

$$\frac{\omega^2}{k^2} = \frac{1}{2\rho}\left[(C_{11}+C_{44}) \pm (C_{12}+C_{44})\right]$$

Two solns, either

$$\boxed{\frac{\omega}{k} = \sqrt{\frac{1}{2\rho}(C_{11}+C_{12}+2C_{44})}}$$

or

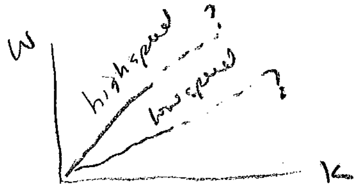
$$\boxed{\frac{\omega}{k} = \sqrt{\frac{1}{2\rho}(C_{11}-C_{12})}}$$

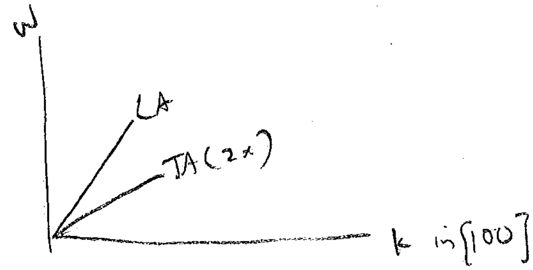
To determine direction of oscillation for these velocities, plug that $\frac{\omega}{k}$ value back into Eqn 3.57 and solve for $\frac{u_y}{u_x}$ ratio. One turns out to be $u_y = u_x$, the other $u_y = -u_x$.

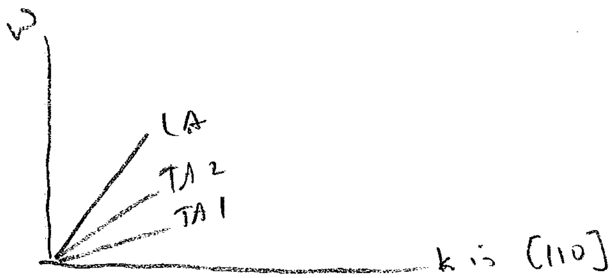
Dispersion relation

$v = \frac{\omega}{k}$ → only true for long λ (small k)

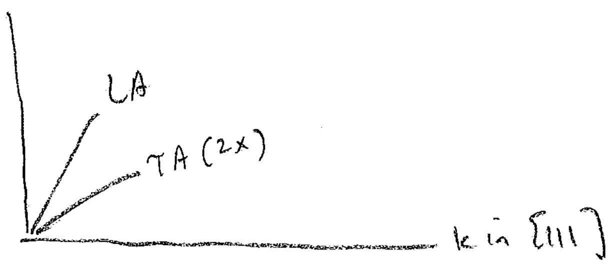
"acoustic waves"







With computers could likely do arbitrary direction in BZ $\omega(\vec{k})$



Done w/ Ch 3

skipped "Elastic Energy Density" pg 77-78

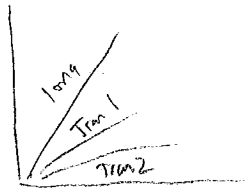
and "Bulk Modulus & Compressibility" pg 80

Chapter 4: Phonons

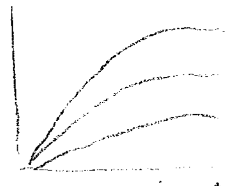
So far we've focused on long wavelength oscillations (small k)

There ω vs k is straight line, and slope is speed of elastic wave (HW problem 3.2)

Figure 11
pg 101



what about larger k?

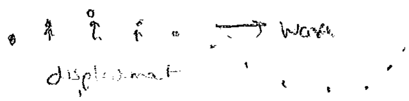


why is there a k_{max}?

Now we'll answer some of these questions

Basic answer: leaving the continuum limit!

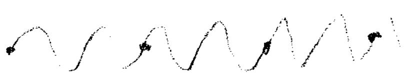
Consider $\lambda = 10a \rightarrow k = \frac{2\pi}{10a} = \frac{\pi}{5a}$



But what if $\lambda = a$? $k = 2\pi/a$

is there a wave?

$\lambda = a/2$ $\quad k = 4\pi/a$



That's why there's a k_{max}! → for $\lambda < a$ doesn't make sense

$\boxed{\text{Fig 5 pg 93}}$ solid curve contains no more info than dashed curve
only k in 1st BZ are needed!