$a = \frac{\Delta v}{\Delta t} \simeq \frac{v_d}{\tau}$ in between collisions

Also $\Sigma F = ma \rightarrow -eE = m\frac{v_d}{\tau}$

$v_d = \frac{-e\tau}{m} E$

electron accelerates for a bit, but then scatters in some random direction (resetting velocity)

Net effect: some overall "drift velocity", $v_{drift} \sim E$

$$\frac{\text{current}}{\text{area}} = \left(\frac{\text{electrons}}{\text{volume}}\right)\left(\frac{\text{charge}}{\text{electron}}\right)\left(\frac{\text{length}}{\text{time}}\right)$$

$$J = n(-e)\, v_{drift}$$

$v_{drift} = -\frac{e\tau}{m} E$ from above

$$\text{so } J = \frac{ne^2\tau}{m} E$$

$$J = \sigma E$$

σ → electrical conductivity

Back to thermal conductivity

Back in Phys 123: $\frac{Q}{t} = \frac{k A \Delta T}{\text{length}}$ (k = thermal conductivity)

Kittel: $\underbrace{\frac{Q}{At}}_{\text{"}j_u\text{" flux of thermal energy}} = -k \frac{dT}{dx}$

($-$: so flow is in right direction)

Where does it come from? Same thing: phonon scattering

if no scattering, get "ballistic transport", leads to very different situation

Scattering:

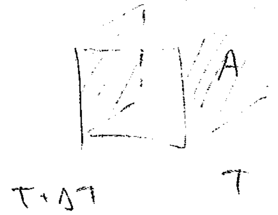
τ = ave scattering time (assume it exists)

λ = mean free path (distance between scattering events)

$$v_{rms} = \frac{\lambda}{\tau}$$

Assume τ = independent of phonon energy day 20 pg 2

Δx distance between scattering events $= v_x \tau$



$n = \frac{\#\ \text{phonons}}{\text{volume}}$ (like n for electrons)

poor label, but what it uses

$n v_x = \frac{\#}{\text{vol}} \cdot \frac{\text{length}}{\text{time}} = \frac{\#\ \text{phonons}}{\text{area} \times \text{time}}$ = "phonon flux"

maybe do these in reverse order since j_u is on previous page

Energy transport $\tilde{C}_v$ = heat cap. for single phonon $= \frac{C_v}{N}$

ie. $Q = \tilde{C}_v \, \Delta T$

$j_u = \frac{\text{energy lost from box}}{\text{area} \times \text{time}} = -(n v_x)(\tilde{C}_v \Delta T)$

$\frac{\#\ \text{phonons}}{\text{area} \cdot \text{time}}$ $\frac{\text{energy}}{\text{ph}}$

thermal energy flux

Also: ΔT = gradient Δx

$\frac{dT}{dx} \times \Delta x$

$\Delta T = \frac{dT}{dx} \times (v_x \tau)$ $= \Delta x$, τ = time between scattering events

$j_u = -n v_x^2 \tilde{C}_v \tau \frac{dT}{dx}$

$= -\frac{1}{3} n v^2 \tilde{C}_v \tau \frac{dT}{dx}$

$\langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle$

$v\tau = l$ (use) mean free path

$n \tilde{C}_v = \frac{\text{phonons}}{\text{volume}} \cdot \frac{\text{heat cap}}{\text{phonon}}$

$= C = \frac{\text{heat cap}}{\text{volume}}$ ie $Q = VC\Delta T$

poor symbol!

$j_u = -\frac{1}{3} C v l \frac{dT}{dx}$

compare w/ previous page, means that

$$\boxed{K_{\text{thermal}} = \frac{1}{3} C v l}$$

Table 2 pg 122 some representative values of C, K, and l (for $v = 5000\ \text{m/s}$ as typical velocity)

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What contributes to scattering?

- defects / surface → side note: in diamond, ℓ is limited by isotopes, which seem to be "defects" to the phonons called "isotope effect"
- electrons
- other phonons
- ...

Case 1: low temp, recall $C \sim T^3$

- not many other phonons present, ℓ is limited by defects/surface, Expect ℓ independent of temp

Then $K = \frac{1}{3} C v \underbrace{\ell}_{\text{const}}$

$K \sim C$

expect $\underline{K \sim T^3}$ seen in Fig 18, 19 pg 127 NaFl, Ge

Case 2 high temp, recall $C \sim$ const. But ℓ is no longer const.
phonon-phonon scattering $\ell^{-1} \sim n^2$ (prob. of 2 phonons)

2 phonons collide + merge (3 phonon event) or ← G ← recip lattice vector to bring back to 1BZ called "Umklapp processes"

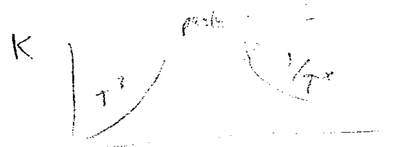
$$n = \frac{1}{e^{\hbar\omega/kT} - 1}$$

↳ if T is high, this is small

$$= \frac{1}{\cancel{1} + \frac{\hbar\omega}{kT} \cancel{-1}} = \frac{kT}{\hbar\omega}$$

$\ell \sim \frac{1}{T^x}$, $x \approx 2$, possibly down to 1 from experiments

$$K = \frac{1}{3} C v \ell \Rightarrow K \sim \frac{1}{T^x}$$



Again, seen in Fig 18, 19 pg 127

~~Not quite "high" → then Boltzmann factor governs how many phonons have high energy, thus $n \sim e^{-\text{const}/T}$ $K \sim \ell \sim \frac{1}{n^2} \sim e^{\text{const}/T}$~~

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Chapter 6 Electrons!

plan of attack

- Ch 6 "Free electron Fermi gas" - no crystal structure (mostly explains metals)
- exam 2 → Ch 7 "Nearly free electron model" - bare minimum structure (periodic)
- Ch 7, cont. more complete band structure
- Ch 8 Semiconductors (and insulators)
  ↳ semiconductors that just don't know it yet :)

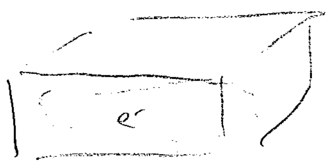
~~[illegible]~~

Then odds + ends - optical properties

- etc.

Tyler & me out of town
Mon = Dr VanSlyke
Wed = Dr Davis

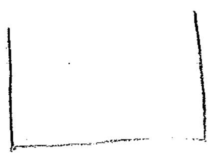
Free electrons - Lorentz quote on pg 133



electrons confined in a box (the crystal)

Intro

Phys 222



x=0 x=L

infinite square well

$H\psi = E_n \psi$

$H \to -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

(Eigenvalue eqn)

results: $\psi_n = A \sin \frac{\pi n x}{L}$ are wavefunctions (eigenfunctions)

satisfies $\psi(x=0) = 0$ and $\psi(x=L) = 0$

Plug into eqn $-\frac{\hbar^2}{2m}\left(-\left(\frac{\pi n}{L}\right)^2\right)\psi = E\psi$

$$\boxed{E = \frac{\hbar^2}{2m}\frac{n^2\pi^2}{L^2}}$$

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day 20 #5

But note $\sin\frac{n\pi x}{L}$ → $k = \frac{n\pi}{L}$

$$\boxed{E = \frac{\hbar^2 k^2}{2m}}$$

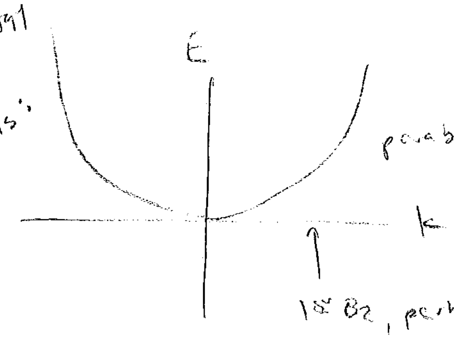
compare $E = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

if $\boxed{p = \hbar k}$ then that energy eq. makes sense

same as momentum of wave

electron energy vs k

roughly this:



Note: for phonons dispersion curve repeated because of lattice periodicity

Here we haven't made that assumption yet. (But we will!)

(And same result!)

↓

- periodic dispersion,
- finite points separated by $2\pi/L$ (crystal dimension)

(end of intro)

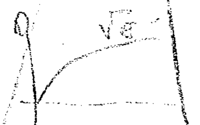
Density of States

3-D $D(E) = \left(\underset{\uparrow\ \text{spin states}}{2}\right)\frac{4\pi k^2}{dE/dk}\,\frac{1}{\left(\frac{2\pi}{L}\right)^3}$

$E = \frac{\hbar^2 k^2}{2m} \rightarrow \frac{dE}{dk} = \frac{2\hbar^2 k}{2m} = \frac{\hbar^2}{m}\sqrt{\frac{2mE}{\hbar^2}}$

$D(E) = \frac{2\cdot 4\pi\left(\frac{2mE}{\hbar^2}\right)}{\frac{\hbar^2}{m}\sqrt{\frac{2mE}{\hbar^2}}} \times \frac{V}{(2\pi)^3}$

$$\boxed{D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}}$$



double check by Kittel

factors of 2: $\frac{16}{\sqrt{2}\,8} = \frac{2}{\sqrt{2}} = \sqrt{2}$ || $\frac{2^{3/2}}{2} = 2^{1/2} = \sqrt{2}$ ✓ yes, the same