Some numbers...

$a = 4.225$ Å

Sodium

1 valence electron / atom

$n = \frac{N_{electrons}}{V} = \frac{N_{atoms}}{V} = 2.652 \cdot 10^{28} / m^3$ from Table 1.4, pg 21

Let's say $m_{electron} = 9.11 \cdot 10^{-31}$ kg (actually we'll find that mass often acts like it's reduced. It doesn't require as much force to produce acceleration as you might think)

Then $E_F = \left(3\pi^2 \cdot 2.652 \cdot 10^{28}\ m^{-3}\right)^{2/3} \cdot \frac{(1.055 \cdot 10^{-34}\ Js)^2}{2 \cdot 9.11 \cdot 10^{-31}\ kg}$

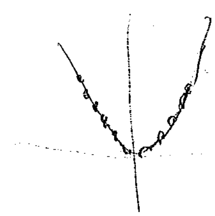
$= 5.199 \cdot 10^{-19}\ J \times \frac{1\ eV}{1.6 \cdot 10^{-19}\ J}$

$= \boxed{3.25\ eV}$

compare to $k_B T = (1.38 \cdot 10^{-23})(300)$ at room T

$= 4.14 \cdot 10^{-21}\ J$

$= \boxed{.0258\ eV}$ the assumption of T close to 0 is justified!



$\leftarrow E_F$

only electrons very close to here can even absorb kT of energy (deep electrons can't move up in energy because nearby states are filled)

$E^2 = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

$k_F = \frac{1}{\hbar}\sqrt{2mE_F}$

$= \frac{1}{1.055 \cdot 10^{-34}}\sqrt{2 \cdot 9.11 \cdot 10^{-31} \cdot 5.199 \cdot 10^{-19}}$

$= 9.23 \cdot 10^{9}\ m^{-1} \times \frac{1 m}{10^{10}\ Å}$

$\boxed{k_F = .923\ Å^{-1}}$

compare BZ edge

$k_{edge} \approx \frac{\pi}{a} = \frac{\pi}{4.225 Å} = \boxed{.744\ Å^{-1}}$

$$\cancel{\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3}\right]^{1/2}}$$

$$\cancel{\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{\hbar}{\sqrt{2m}}\left(3\pi^2\frac{N}{V}\right)^{1/3}}$$

$$\cancel{\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E_F^{1/2} = \frac{3N}{2E_F}}$$

~~Also $E_F = \left(\frac{\hbar^2}{2m}\right)^{3/2}$~~

$$D(E_F) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{1/2} \longrightarrow \text{want to simplify}$$

$$\text{Also } E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3}$$

from yesterday or earlier today

solve for V

$$\left(\frac{2mE_F}{\hbar^2}\right)^{3/2} = 3\pi^2\frac{N}{V}$$

$$V = 3\pi^2 N\left(\frac{2mE_F}{\hbar^2}\right)^{-3/2}$$

"and then a miracle occurs"

$$D(E_F) = \frac{\left[3\cancel{\pi^2}N\left(\frac{\cancel{2m}E_F}{\cancel{\hbar^2}}\right)^{-3/2}\right]}{2\cancel{\pi^2}}\left(\cancel{\frac{2m}{\hbar^2}}\right)^{3/2} E_F^{1/2}$$

$$\boxed{D(E_F) = \frac{3}{2}\frac{N}{E_F}}$$

useful simplified form as you'll see in a minute

end of step (c)

But this is now not a function of E

that is $D(E) \neq \sim \frac{1}{E}$

Back to calculation of U

$$U = \int E\, D(E)\, f(E)\, dE$$

$D(E)$: approximate this as $D(E_F)$, take out of integral

$f(E) = \dfrac{1}{e^{+(E-\mu)/kT}+1}$; μ: approximate this as E_F, a constant

$$\boxed{U \approx D(E_F) \int_0^{\infty} \frac{E\, dE}{e^{(E-E_F)/kT}+1}}$$

this works because f changes mostly around E_F, so when we take $\frac{\partial U}{\partial T}$ later, the only nonzero part will be when $E \approx E_F$

end of step (d)

$C = \frac{\partial U}{\partial T}$ → take $\frac{\partial}{\partial T}$ inside integral

$$\frac{\partial}{\partial T}\left(\frac{1}{e^{(E-E_F)/kT}+1}\right) = \frac{-1}{\left(e^{(E-E_F)/kT}+1\right)^2} \times e^{(E-E_F)/kT} \cdot \left(-\frac{(E-E_F)}{kT^2}\right)$$

$$C = D(E_F)\, \frac{1}{kT^2} \int_0^{\infty} \frac{(E-E_F)\, E\, e^{(E-E_F)/kT}}{\left(e^{(E-E_F)/kT}+1\right)^2}\, dE$$

let $x = \frac{E-E_F}{kT}$ → $E = kTx + E_F$, $dE = kT\,dx$

$$\boxed{C = D(E_F)\, k \int_{-E_F/kT}^{\infty} \frac{x e^x}{(e^x+1)^2}\, (kTx + E_F)\, dx}$$

end of step (e)

small T approx: take integral from $-\infty$ to $+\infty$, then this part integrates to 0 since $\frac{xe^x}{(e^x+1)^2}$ is odd

$$C = D(E_F)\, k^2 T \underbrace{\int_{-\infty}^{\infty} \frac{x^2 e^x}{(e^x+1)^2}}_{=\pi^2/3}$$

$$= \left(\frac{3}{2}\frac{N}{E_F}\right)\left(k^2 T\right)\left(\frac{\pi^2}{3}\right)$$

$$\boxed{C_{el} = \frac{\pi^2}{2}\frac{k^2 T}{E_F} N}$$

$C \sim T$

C_{el}: from electrons

Combined w/ phonons

$$C = \underbrace{\gamma T}_{\text{electron}} + \underbrace{A T^3}_{\text{phonon}} \quad \text{at low } T$$

$$\underbrace{\frac{C}{T}}_{y \text{ axis}} = \gamma + A \underbrace{T^2}_{x \text{ axis}}$$

Fig 9 for potassium pg 145

$$y = b + mx$$

Fits perfectly!

Table 2 pg 146 - Experimental γ vs Theoretical γ

→ pretty close, but some differences.

Attributed differences to "(thermal) effective mass"

Trace back... where's the m?

$C \sim D(E_F)$

$\sim m^{3/2} E_F^{1/2}$

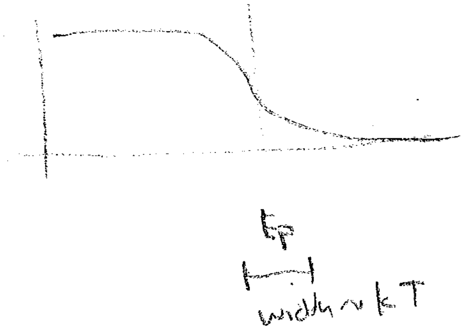
$E_F \sim \frac{1}{m}$, so this $\sim \frac{1}{m^{1/2}}$

$C \sim m$

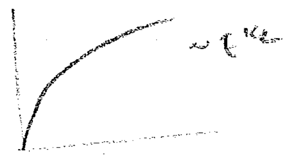
Check on this
maybe $C \sim \frac{1}{m}$
since $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{2/3}$

Qualitative calculation of C_V

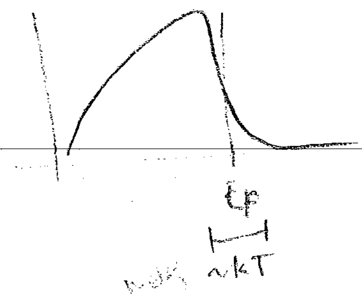
At low T's 1) $f(E)$ looks like



2) $D(E) =$



Multiply together



how many states there? (that are involved in "action")

$$= N \times \frac{kT}{E_F}$$

if each one gains kT,

then

$$U = N \cdot \frac{k^2 T^2}{E_F}$$

$$\boxed{C_V = 2\frac{Nk^2 T}{E_F}}$$

compare to $\underbrace{\frac{\pi^2}{2}}_{\text{constant a little off}} \underbrace{N\frac{k^2 T}{E_F}}_{\text{right on!}}$

"Fermi surface"
→ more in Ch 9
(which we will not really discuss)

(out of town Mon & Wed)