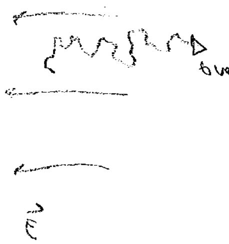
day 23 / day 24 — out of town   Day 25   pg 1

Electrical Conductivity

Classical scattering

Review: did this last chapter (day 20)



$$a = \frac{\Delta v}{\Delta t} = \frac{v_d}{\tau}$$

← because y + z directions will average to zero, and x direction will average to v_d

also $\Sigma \vec{F} = m\vec{a}$

$$a = \frac{-e\vec{E}}{m}$$

$$\underline{\underline{v_d = \frac{-eE}{m}\tau}}$$

$$J = \frac{\text{current}}{\text{area}} = \frac{\text{charge}}{\text{area}\cdot\text{time}} = n\,(-e)\,v_d$$

J → "charge flux", I guess

n → $\frac{\text{electrons}}{\text{area}\cdot\text{length}}$, $(-e)$ → $\frac{\text{charge}}{\text{electron}}$, v_d → $\frac{\text{length}}{\text{time}}$

$$\boxed{J = \underbrace{\frac{n e^2 \tau}{m}}_{\sigma} E}$$

values of σ in Table 3 pg 149

Ohm's Law! $J = \sigma E$

Warning about units: Table 3 looks like cgs, but is really SI pg 149

$$\frac{I}{A} = \sigma\,(V/\ell) \rightarrow V = I \times \text{constant}$$

$$R = \frac{1}{\sigma}\frac{\ell}{A}$$

$$R = \rho\frac{\ell}{A}$$

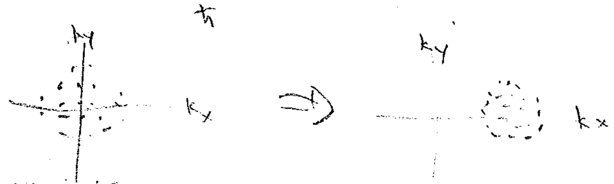
familiar?

Discrete k values

$$\Sigma F = m\frac{d\vec{v}}{dt} \rightarrow = \frac{d\vec{p}}{dt} = \hbar\frac{d\vec{k}}{dt}$$

$$-e\vec{E} = \hbar\frac{d\vec{k}}{dt}$$

$$\vec{k} = \vec{k_0} - \frac{e\vec{E}}{\hbar}t$$



scatterings: k_x doesn't get arbitrarily big but reaches some steady-state value

Things match with picture above:

$$k = k_0 - \underbrace{\frac{e\vec{E}}{\hbar}\tau}_{\text{constant offset}}$$

(breaks down for large E)

∴ Electrical conduct, $\sigma = \frac{n e^2 \tau}{m}$, values in Table 3 on pg 149

Recall last chapter (day 20)

thermal cond. of phonons $k_{th} = \frac{1}{3} C v \ell$ $\longrightarrow$ "heat cap" / Volume

values in Table 1 pg 116

→ model: scattering / mean free path

What about thermal cond. of electrons?

Same result

$$k_{el} = \frac{1}{3} \underbrace{C_{\frac{\text{electrons}}{\text{Vol.}}}} \, v_{\text{Fermi}} \, \ell_{\text{electrons}} \longrightarrow v_F \cdot \tau$$

since only electrons on Fermi surface can change energy $\frac{1}{2} m v_F^2 = E_F$

from a couple of days ago, (+handout)

$$C_V = \frac{\pi^2}{3} D(\epsilon_F) k_B^2 T$$

$$= \frac{\pi^2}{3} \left[\frac{3}{2} \frac{N}{E_F}\right] k_B^2 T$$

Combine

$= v_F^2 \tau$

$= \frac{2 E_F}{m} \tau$

$$k_{el} = \frac{1}{3} \left(\frac{\pi^2}{2} \frac{N}{\cancel{E_F}} k^2 T \right] \frac{1}{V} \left[\frac{2 \cancel{E_F}}{m} \tau \right]$$

$$\boxed{k_{el} = \frac{\pi^2}{3} n \frac{k^2 T}{m} \tau}$$

ratio: $$\frac{k_{el}}{\sigma_{el}} = \frac{\frac{\pi^2}{3} \cancel{n} \frac{k^2 T}{\cancel{m}} \cancel{\tau}}{\cancel{n} \frac{e^2 \cancel{\tau}}{\cancel{m}}} = \boxed{\underbrace{\frac{\pi^2}{3} \frac{k_B^2}{e^2}} T}$$

Wiedemann-Franz Law

$\frac{k_{thermal}}{\sigma} \sim T$

"Lorentz number"

proportionality constant.

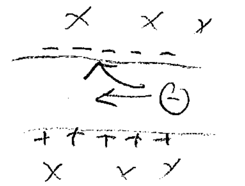
No n! No m!

Independent of material!

$\Big\} = 2.45 \cdot 10^{-8} \frac{W \Omega}{K^2}$

(mostly) See Table 5 pg 157, for experimental ratios all very close to this number.

Last item from chapter: magnetic fields



transverse voltage

but how strong?

$$\vec{F} = q\vec{E} + q\,\vec{v}\times\vec{B} \quad \text{(Phys 220)}$$

$$\vec{F} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

also assume a damping term $\sim \vec{v}$ (like air resistance)

$$\Sigma F = m a \longrightarrow m\frac{\partial \vec{v}}{\partial t} = -e\left(\vec{E}+\vec{v}\times\vec{B}\right) - \frac{m\vec{v}}{\tau}$$

← characteristic damping time

Steady state: $\frac{\partial}{\partial t} \to 0$

$$\frac{m\vec{v}}{\tau} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

Typically $\vec{E} \perp \vec{B}$ in "Hall Effect" experiments

$\vec{E}$ in x-y plane; $\vec{B}$ in $\hat{z}$

v also in x-y plane

units of frequency: ω_c "cyclotron"

$$\frac{m v_x}{\tau} = -e\left(E_x + v_y \underbrace{B_z}_{\text{just "B"}}\right) \longrightarrow v_x = -\frac{e\tau}{m}E_x - \left(\frac{eB}{m}\right)\tau v_y$$

and

$$\frac{m v_y}{\tau} = -e\left(E_y - v_x B_z\right) \longrightarrow v_y = -\frac{e\tau}{m}E_y + \left(\frac{eB}{m}\right)\tau v_x$$

part of matrix eqn for HW

Now, force $v_y = 0$ for current like

$$v_x = -\frac{e\tau}{m}E_x \quad ; \quad 0 = -\frac{e\tau}{m}E_y + \left(\frac{eB}{m}\right)\tau v_x$$

$$0 = -\frac{\cancel{e}\cancel{\tau}}{\cancel{m}}E_y + \left(\frac{\cancel{e}B}{\cancel{m}}\right)\cancel{\tau}\left(-\frac{e\tau}{m}E_x\right)$$

$$\boxed{E_y = -\frac{e\tau}{m}B\,E_x}$$

Transverse field! leads to "Hall voltage", what you actually measure

negative number because $e = +1.6\times10^{-19}$ C i.e. $q = -e$