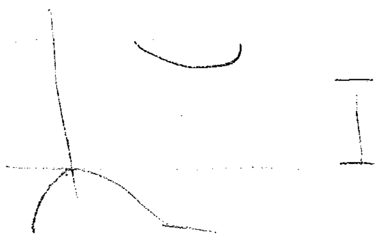
Ch. 8
Effective Mass



real — band gap opens up due to crystal potential
- near zone edges, it's quadratic.

 ← put electron at bottom of slope, how does it behave?

Free electron $E = \frac{\hbar^2 k^2}{2m} \longrightarrow \frac{dE}{dk} = \frac{\hbar^2}{m} k$

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$$

Electron in crystal potential $\boxed{\frac{1}{m_{eff}} = \frac{1}{\hbar^2} \frac{d^2E}{dk^2}}$

Notation: $m_{eff} = m^*$

rigorously form
$\vec{p} = \hbar \vec{k}$
$\vec{F} = \hbar \frac{d\vec{k}}{dt}$
+ wave prop velocity

Semiconductors: $m_{eff} \simeq .01 - .1\, m_{electron}$ (near band gap)

- Newton's 2nd Law not violated for crystal as a whole
- It means when electron has force on it by e.g. applied E or B field, it gets accelerated more than you would expect because of forces also applied to it by the ions (nuclei) that are causing the overall periodic potential

tensor

If curvature is different in different directions, then need tensor

$$\left(\frac{1}{m}\right)_{xx} = \frac{1}{\hbar^2} \frac{d^2E}{dk_x^2}$$

$$\left(\frac{1}{m}\right)_{xy} = \frac{1}{\hbar^2} \frac{d^2E}{dk_x dk_y}$$

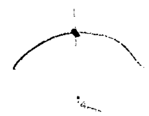
etc.

Then N2 is $F = m\frac{dv}{dt} \longrightarrow \frac{dv}{dt} = \frac{1}{m} F \longrightarrow \frac{dv_x}{dt} = \left(\frac{1}{m}\right)_{xx} F_x + \left(\frac{1}{m}\right)_{xy} F_y + \left(\frac{1}{m}\right)_{xz} F_z$

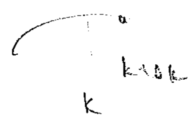
$\frac{dv_y}{dt}$ ($x \to y$ in each subscript)

etc.

m_{eff} can even be negative!



What it means: going from k to k+δk



the lattice causes a bigger $\frac{d\vec{p}}{dt}$ than the applied force does

Convert old eqns

$$\sigma = \frac{ne^2\tau}{m} \rightarrow \sigma = \frac{ne^2\tau}{m^*}$$ small m → better conductor

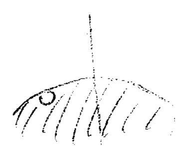
$$\omega_c = \frac{eB}{mc} \rightarrow \omega_c = \frac{eB}{m^*c}$$ one way to measure m^* is cyclotron resonance

$$D(E) = \frac{V}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{E}$$

$$E_F = \left(\frac{3\pi^2 N}{V}\right)^{2/3}\left(\frac{\hbar^2}{2m^*}\right)$$ X

etc.

Holes — like auditorium w/ one empty seat. (also like bubbles.)



electron picture
m_e = negative

⟹ hole picture

E_h

$$E_h = \frac{\hbar^2 k_h^2}{2m_h}$$

m_h = positive

$m_h = -m_e$

$\boxed{\vec{k}_h = -\vec{k}_e}$

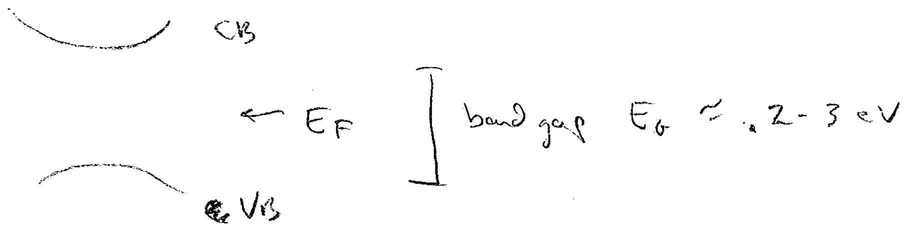
$E_h = -E_e$ → band flipped upside down

Charge = $+|e|$

Spin = also reversed

people frequently talk of holes, but don't usually draw valence bands like ∪

Valence + Conduction bands



General Rule (Problem 9.3) – $m^* \sim E_g$, approximately for direct gap materials

:1 may or may not assign, uses 2nd order perturb. theory, uses bracket notation, would need to translate

Table of E_g: pg 190 Table 1

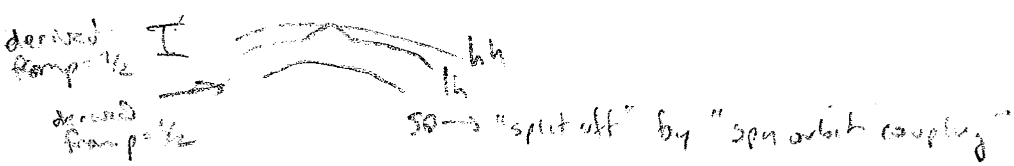
Table of m^*: pg 201 Table 2

handout of Band gap Engineering? Figure? 1 Color?

VB details

See Fig 14 pg 205 for Ge

or handout for Si + GaAs?



← how many electrons actually up here?

$$f = \frac{1}{e^{(E-\mu)/kT} + 1}$$

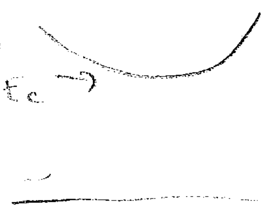
"tail of Fermi" $E - \mu >> kT$

$$f \approx \frac{1}{e^{(E-\mu)/kT}}$$
$$= e^{-(E-\mu)/kT}$$

Side note, maybe after next class

CB

E_c →



← density of states:

just like free electron except

→ m^* to explain different curvature

→ $E - E_c$ to explain vertical offset ←

as done in Chap 6 handout: $\frac{D(E)}{V} = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}$

concentration $n = \int_{E_c}^{\infty}\left(\frac{D(E)}{V}\right) f(E)\, dE$

$$= \int_{E_c}^{\infty}\left(\frac{1}{2\pi^2}\right)\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}\, e^{-(E-\mu)/kT}\, dE$$

$$= \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} e^{+\mu/kT}\underbrace{\int_{E_c}^{\infty}(E-E_c)^{1/2}\, e^{-E/kT}\, dE}_{\text{Mathematica} \;\; \frac{\sqrt{\pi}}{2} e^{-E_c/kT}(kT)^{3/2}}$$

Eqn 8.39 pg 206

$$\boxed{n = \underbrace{2\left(\frac{m^* kT}{2\pi\hbar^2}\right)^{3/2}}_{n_0} e^{-(E_c-\mu)/kT}} = n_0 e^{-(E_c-\mu)/kT}$$

$n_0 = 2\left(\frac{m^* kT}{2\pi\hbar^2}\right)^{3/2}$ pg 214

proof of 2's:

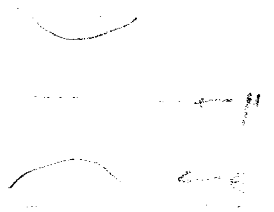
$2 \cdot \frac{2^{3/2}}{2^{3/2}}$ also $\frac{2^{3/2}}{2}\cdot\frac{1}{2} = 2^{3/2}\, 2^{-2}$

$= 2^{-1/2}$ ✓

problem: don't know μ.

Do same thing for holes in valence band

$$f_h(E) = 1 - f(E)$$
$$= 1 - \frac{1}{e^{(E-\mu)/kT}+1} = \frac{e^{(E-\mu)/kT}}{e^{(E-\mu)/kT}+1}$$



$$\frac{e^x}{e^x+1} \cdot \frac{e^{-x}}{e^{-x}} = \frac{1}{1+e^{-x}}$$
$$= \frac{1}{e^{(\mu-E)/kT}+1}$$
$$\approx e^{-(\mu-E)/kT}$$

$\mu - E$ = positive

(E = energy of electrons, really not energy of holes which would have opposite sign)

Same integral — Typo pg 206

$$p = \int_{-\infty}^{E_v} \frac{D_h(E)}{V}\, f_h(E)\, dE$$

modified for holes

pg 206
Eqn 8.42
Typo in Kittel!

$$p = \underbrace{2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}}_{\text{could be called } p_0 \text{, but isn't in Kittel. I'll use that symbol later, though}} e^{-(\mu - E_v)/kT} \quad \text{(Same integral)}$$

Multiply together

day 34 pg 1

$$np = 4\left(\frac{kT}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} e^{-\frac{1}{kT}(E_c - \cancel{\mu} + \cancel{\mu} - E_v)}$$

$$\boxed{np = \underbrace{4\left(\frac{kT}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2}}_{n_0 p_0} e^{-E_g/kT}}$$

only assumption: μ far from both CB & VB

could be "intrinsic", could be "doped"

↳ but careful b/c 1st condition is still met "uncompensated"