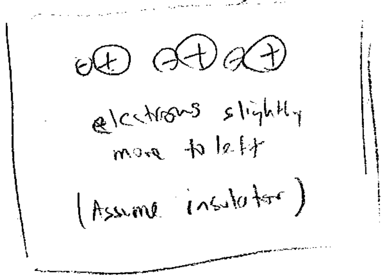
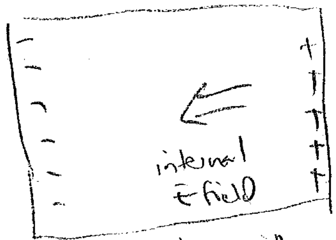
Inside materials $\underrightarrow{E}$



overall end result



which partially cancels external field

For Maxwell's Eqns, need total $\vec{E} = \vec{E}_{ext} + \vec{E}_{int}$

instead, use Polarizability $\vec{P} = \epsilon_0 \chi_e \vec{E}$

↳ electric susceptibility. Describes dipole moments that get formed ($=$ ave $\frac{\text{dipole moment}}{\text{volume}}$)

"D-field" or "displacement field" $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

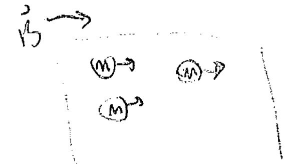
then

① $\boxed{\vec{\nabla} \cdot \vec{D} = \rho_{extra}}$ ← "ρ_{free}"

② $\boxed{\vec{\nabla} \cdot \vec{B} = 0}$ still

③ $\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ still (why?)

Same sort of deal for B field



$\vec{M} = \chi_m \vec{H}$

$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ } historically done this way instead of $\vec{M} = \chi_m \vec{B}$ and $\vec{H} = \mu_0 \vec{B} + \vec{M}$

Magnetic moment / volume

Then $\boxed{\nabla \times \vec{H} = \vec{J}_{extra} + \frac{\partial \vec{D}}{\partial t}}$

Most materials <u>nonmagnetic</u>. Then $\vec{M} = 0$
$\vec{H} = \frac{\vec{B}}{\mu_0}$

Last eqn is $\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}_{extrn} + \mu_0 \frac{\partial \vec{D}}{\partial t}}$

<u>Dielectric function</u> Write $\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E}$

ϵ: permittivity; ϵ_r: dielectric const, relative permittivity

$(= \epsilon_0 E + P$
$= \epsilon_0 E + \epsilon_0 \chi_e E$
$= \epsilon_0 (1+\chi_e) E$
so $\epsilon_r = 1+\chi_e)$

If no extra charges/currents:

$$\boxed{\begin{aligned} &(1) \quad \nabla \cdot \vec{E} = 0 \\ &(2) \quad \nabla \cdot \vec{B} = 0 \\ &(3) \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &(4) \quad \nabla \times \vec{B} = \epsilon_0 \epsilon_r \mu_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

Speed of Light:

$\nabla \times (3)$: $\quad \nabla \times (\nabla \times E) = -\frac{\partial}{\partial t} (\nabla \times B)$

$$\vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{E}}_{0}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\epsilon_0 \epsilon_r \mu_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\boxed{\nabla^2 \vec{E} = \epsilon_0 \epsilon_r \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Wave eqn!

1D: $\quad \frac{\partial^2 E}{\partial x^2} = \epsilon_0 \epsilon_r \mu_0 \frac{\partial^2 E}{\partial t^2}$

try $E = E_0 \cos(kx - \omega t) = E_0 e^{i(kx-\omega t)}$

$-k^2 E = \epsilon_0 \epsilon_r \mu_0 (-\omega^2) E$

$\frac{\omega^2}{k^2} = \frac{1}{\epsilon_0 \epsilon_r \mu_0}$

$$\boxed{v = \sqrt{\frac{1}{\epsilon_0 \epsilon_r \mu_0}} = \frac{c}{n}}$$

From $B_y = B_0 \cos(kx - \omega t) = B_0 e^{i(kx - \omega t)}$

get same result,

and also $\boxed{B_0 = \frac{1}{V} \cdot E_0} = \frac{1}{c/n} E_0$

test: in space $\epsilon_r = 1$

$$\frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{8.85 \cdot 10^{-12} \cdot 4\pi \cdot 10^{-7}}} = 2.9986 \cdot 10^8 \frac{m}{s}$$

Back in 123 waves on a string, found

$$r = \frac{V_2 - V_1}{V_2 + V_1} = \text{ratio of amplitudes,}$$

$$R = |r|^2 = \text{ratio of intensity / power}$$

Something here!

$$r = \frac{\frac{c}{n_2} - \frac{c}{n_1}}{\frac{c}{n_2} + \frac{c}{n_1}} = \frac{\frac{n_1 - n_2}{\cancel{n_1 n_2}}}{\frac{n_1 + n_2}{\cancel{n_1 n_2}}} = \boxed{\frac{n_1 - n_2}{n_1 + n_2}}$$

or, if air → glass

$$R = \left(\frac{1-n}{1+n}\right)^2 = \left(\frac{n-1}{n+1}\right)^2$$

Measure R → can deduce n

What about absorption? Previous wave equation gave solns like

(this basically taken from Griffiths)

$\cos(kx - \omega t)$

which extends over all space + time + never decays.

Absorption comes from conductivity + friction

electrons $\longleftrightarrow$

+ lose energy due to ohmic heating

conductive: then $\vec{J} = \sigma \vec{E}$ Ohm's Law, already discussed

side note: really $\vec{J} = \overleftrightarrow{\sigma} \vec{E}$
and $\vec{D} = \overleftrightarrow{\epsilon} E$

Then Eqn 4 = $\nabla \times B = \mu_0 (\sigma \vec{E}) + \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t}$

take $\nabla \times (\nabla \times E)$ again ...

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \epsilon_r \frac{\partial^2 E}{\partial t^2} + \mu_0 \sigma \frac{\partial E}{\partial t}}$$

damping!

Looking ahead ... solns now like

$e^{-Kz} \cdot \cos(kz - \omega t)$

How to represent? Still can use complex numbers

$e^{-Kz} \cdot e^{i(kz - \omega t)}$

$e^{i((k + iK)z - \omega t)}$

write $\tilde{k} = k + iK$

complex wave number!

Just a math. trick to represent damping

Now guess $\tilde{E} = \tilde{E}_0 e^{i(\tilde{k}z - \omega t)}$

Wave eqn: $-\tilde{k}^2 \tilde{E} = \mu_0 \epsilon_0 \epsilon_r (-\omega^2) \tilde{E} + \mu_0 \sigma (-i\omega) \tilde{E}$

$$\tilde{k}^2 = \mu_0 \epsilon_0 \epsilon_r \omega^2 + (\mu_0 \sigma \omega) i$$

To find $\tilde{k}$, need to take square root.

How to do square root of eg $3+4i$?

write in polar form

$3+4i = r e^{i\theta}$ $\quad r = \sqrt{3^2+4^2}$ $\quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$= 5 e^{i(53.13^\circ)}$

Then $\sqrt{5 e^{i\,53.13^\circ}} = \sqrt{5}\, e^{i\left(\frac{53.13^\circ}{2}\right)}$

$= \sqrt{5}\cos\left(\frac{53.13^\circ}{2}\right) + \sqrt{5}\, i \sin\left(\frac{53.13^\circ}{2}\right)$

Do that for the $\tilde{k}^2$ equation above.

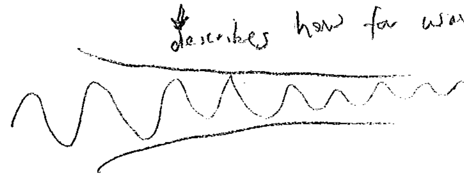
Result:

$$\tilde{k} = \omega\sqrt{\frac{\epsilon_0 \epsilon_r \mu_0}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon_0 \epsilon_r \omega}\right)^2} + 1\right]^{1/2}$$

$$+ i\, \omega\sqrt{\frac{\epsilon_0 \epsilon_r \mu_0}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon_0 \epsilon_r \omega}\right)^2} - 1\right]^{1/2}$$

k_{real} $\qquad \kappa$ (kappa)

describes how far wave penetrates (decay constant)



More complex #s?

$v = \frac{\omega}{k} \rightarrow$ use $v = \frac{\omega}{k_{real}}$

$\frac{c}{n} = \frac{\omega}{k} \rightarrow n = \frac{ck}{\omega} \rightarrow$ use $\tilde{n} = \frac{c\tilde{k}}{\omega}$

$$\tilde{n} = n_{real} + i\, n_{imag}$$

"κ" of course ☺

Griffiths: lowercase, Kittel: upper case

Kittel: $\tilde{n} \leftrightarrow N$

Previous eqn : $R = \frac{n-1}{n+1} \rightarrow \frac{\tilde{n}-1}{\tilde{n}+1} \rightarrow \frac{n+ik-1}{n+ik+1}$

$n = \sqrt{\epsilon_r} \longrightarrow \epsilon_r = n^2$

$\longrightarrow \tilde{\epsilon}_r = (\tilde{n})^2$

day 39 pg 1

Kramers-Kronig relations (ch 15 pg 430-433)

Theorem: whenever you have a complex "response function", in order to guarantee the response happens after the stimulus,

ie. $\vec{P}$ ie $\vec{E}$

the real + imaginary components of the function have to be related to each other, specifically, for dielectric const

$$(\epsilon_r(\omega))_{real} = \frac{2}{\pi} P\left[\int_0^\infty \frac{\omega' \times (\epsilon_r(\omega'))_{imag}}{\omega'^2 - \omega^2} d\omega'\right]$$

derivation found in Jackson, I believe

and $$(\epsilon_r(\omega))_{imag} = -\frac{2\omega}{\pi} P\left[\int \frac{(\epsilon_r(\omega'))_{real}}{\omega'^2 - \omega^2} d\omega'\right]$$

"Cauchy Principle value" related to contour integration of complex numbers. That's all I'll say

You measure eg absorption vs ω for a large number of ω's and you can calculate reflectivity.

or vice versa