day 38 pg 5

Previous eqn: $R = \frac{n-1}{n+1} \rightarrow \frac{\tilde{n}-1}{\tilde{n}+1} \rightarrow \frac{n+ik-1}{n+ik+1}$

$n = \sqrt{\epsilon_r} \longrightarrow \epsilon_r = n^2$

$\longrightarrow \tilde{\epsilon}_r = (\tilde{n})^2$

day 39 pg 1

Kramers-Kronig relations (ch 15 pg 430-433)

Theorem: whenever you have a complex "response function", in order to guarantee the response happens after the stimulus (ie $\vec{E}$), (ie. $\vec{P}$)

the real + imaginary components of the function have to be related to each other, specifically, for dielectric const

derivation found in Jackson, I believe

$$(\epsilon_r(\omega))_{real} = \frac{2}{\pi} P\left[\int_0^{\infty} \frac{\omega' \times (\epsilon_r(\omega'))_{imag}}{\omega'^2 - \omega^2} d\omega'\right]$$

and

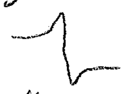
$$(\epsilon_r(\omega))_{imag} = -\frac{2\omega}{\pi} P\left[\int \frac{(\epsilon_r(\omega'))_{real}}{\omega'^2 - \omega^2} d\omega'\right]$$

"Cauchy Principle value" related to contour integration of complex numbers. That's all I'll say

You measure eg absorption vs ω for a large number of ω's and you can calculate reflectivity.

or vice versa

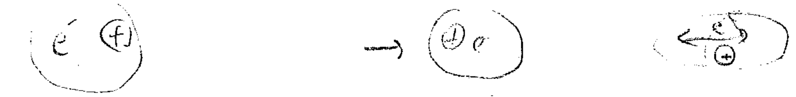
<u>Lorentz model of dielectrics</u> – to explain the UV feature of Fig 16.8 pg 464



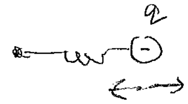
"dielectronic"

mainly from P+W Optics book (Also in Slater Ch16-3)

Apply field   release   bounces back + forth.



kind of like mass on a spring!
(charged mass)



Let's assume there's a resonant freq, ω_0, that describes this.

$$\Sigma F = ma$$

$$m \frac{d^2 \vec{u}}{dt^2} = -m\omega_0^2 \vec{u} + q\vec{E}$$

$\vec{u}$ is displ. from equilibrium

oscillating field from light wave $\vec{E}_0 e^{i(kx-\omega t)}$

Would find an infinite response when $\omega = \omega_0$

... add <u>damping</u>

Simplest model, like air resistance $F_{damping} \sim$ velocity, $\frac{d\vec{u}}{dt}$

$$\boxed{\frac{d^2\vec{u}}{dt^2} = -\omega_0^2 \vec{u} + \frac{q}{m}\vec{E}_0 e^{i(kx-\omega t)} - \gamma \frac{d\vec{u}}{dt}}$$

"damped, driven harmonic oscillator"

Guess solution $\vec{u} = \vec{u}_0 e^{i(kx-\omega t)}$

$$-\omega^2 \vec{u}_0 \cancel{e^{i(kx-\omega t)}} = -\omega_0^2 \vec{u}_0 \cancel{e^{i(kx-\omega t)}} + \frac{q}{m}\vec{E}_0 \cancel{e^{i(kx-\omega t)}} - \gamma(-i\omega)\vec{u}_0 \cancel{e^{i(kx-\omega t)}}$$

$$(\omega_0^2 - \omega^2 - i\gamma\omega)\vec{u}_0 = \frac{q}{m}\vec{E}_0$$

$$\vec{u}_0 = \frac{q}{m}\frac{\vec{E}_0}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$\boxed{\vec{u} = \frac{q}{m}\frac{\vec{E}_0}{\omega_0^2 - \omega^2 - i\gamma\omega} e^{i(kx-\omega t)}}$$

dipole moment $= q \times u$
total " " $= N q \times u$

day 39 pg 3

$$\vec{P} = \frac{\text{total dipole}}{\text{volume}} = \underbrace{\frac{q^2 n}{m}\frac{1}{\omega_0^2-\omega^2-i\gamma\omega}}_{\text{this is } \epsilon_0\chi_e!}\vec{E}_0 e^{i(kx-\omega t)}$$

$P = \epsilon_0 \chi_e E$

$$\epsilon_r = 1 + \chi_e$$

★ $$\boxed{\epsilon_r = 1 + \frac{q^2 n}{\epsilon_0 m}\frac{1}{\omega_0^2-\omega^2-i\gamma\omega}}$$

units of ω_1^2

call it $\boxed{\omega_p = \sqrt{\frac{q^2 n}{\epsilon_0 m}}}$

"plasma frequency"
↳ we'll discuss why this name in another lecture

If more than one resonant frequency, simply sum! (weighted sum actually)
→ ~~eg~~ (eg. because of more than one type of atom)

My title:
"Lorentz permittivity eqn"
Stokes: I don't know
Yu: Sellmeier
Ware: Not Sellmeier

$$\epsilon_r = 1 + \sum_j \frac{f_j\,\omega_{pj}^2}{\omega_{0j}^2-\omega^2-i\omega\gamma_j}$$

weighting factor f_j called "oscillator strength"

when written with λ instead of ω, (for empirical fitting)
called the "Sellmeier Equation"

Wiki: $$n^2 = 1 + \frac{B_1\lambda^2}{\lambda^2-C_1} + \frac{B_2\lambda^2}{\lambda^2-C_2} + \frac{B_3\lambda^2}{\lambda^2-C_3}$$

(used for glass)

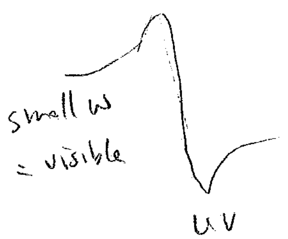
~~Pg~~ ~~$n = \sqrt{\epsilon_r}$ → plot real + imag parts for~~

Simplify eqn for a minute : no damping, just one freq

$$\boxed{\epsilon_r = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}}$$

when ω = small , $\epsilon_r = 1 + \frac{\omega_p^2}{\omega_0^2}$ ← plasma freq ← location of resonance

when ω = big , $\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$



can measure visible index of refraction (ignoring dispersion)

call it n_{opt}

$$n_{opt}^2 = 1 + \frac{\omega_p^2}{\omega_0^2} = \frac{\omega_0^2 + \omega_p^2}{\omega_0^2}$$

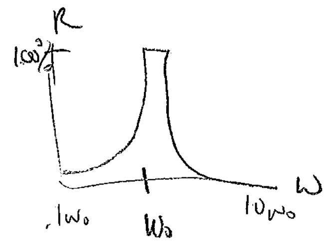
$$\rightarrow \omega_p^2 = (n_{opt}^2 - 1)\,\omega_0^2$$

Watch out for n's!
n = index of refr.
n = # e per volume

$$\boxed{\epsilon_r = 1 + \frac{(n_{opt}^2 - 1)\,\omega_0^2}{\omega_0^2 - \omega^2}}$$ used in Stokes, eqn 16-43 pg 185

$n = \sqrt{(\quad)}$

$R = \frac{n-1}{n+1}$ plot . Stokes Fig 16-6 pg 185



Back to eqn w/ damping

$n = \sqrt{\epsilon_r}$ → plot real + imag. parts of n