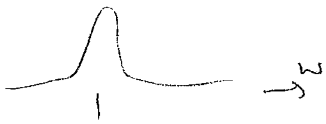
day 40 pg 1

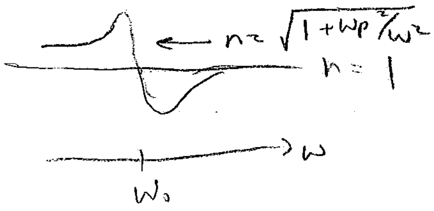
Imag. part of $\hat{n}$



w_0 ← absorption line (for glass, is UV)

exactly as predicted by Kramers Kronig

Real part of $\hat{n}$

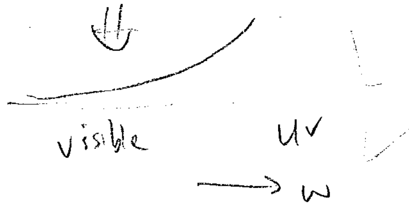


except for this section, "anomalous dispersion"

index always rises w/ frequency

regular "normal dispersion"

$\Downarrow$



<u>Ions</u> – can also act like charged SHOs. Frequencies are much lower

ex NaCl or GaAs 2 atoms/unit cell, 3D

(+ron)(−)

optical phonons: 1 LO, 2 TO

assume degenerate, at W_T

acoustic: 1 LA, 2 TA

$$\epsilon = \epsilon_{phonons} + \epsilon_{electrons}$$

$\epsilon_{electrons} \to \epsilon_\infty$ $(= n_{opt}^2)$ ← Kittel's notation, pg 411

"high freq. dielectric constant"

hi freq: much higher than vibrational freqs, but below electronic excitation energies

$$\epsilon = \epsilon_\infty + \frac{n Q^2}{m \epsilon_0} \frac{1}{w_T^2 - w^2 - i w \gamma}$$

n: # ion pairs / volume

m: reduced mass $\left(\frac{1}{m_1} + \frac{1}{m_2}\right)^{-1}$

Q ≤ valence of ions

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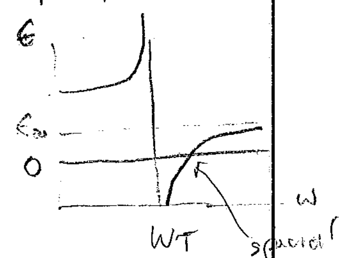
Chang of variables.

Note that when $\omega = \omega$

$$\epsilon = \underbrace{\epsilon_\infty + \overset{\text{basically the plasma freq}^2\ (\omega_P^2)}{\frac{nQ^2}{m\epsilon_0}} \frac{1}{\omega_T^2}}_{\text{"}\epsilon_0\text{"}}$$

☆ Kittel Fig 14-13a, pg 413
(Also Fig 16-7 Stokes pg 187)
☆ plotted for $\gamma = 0$

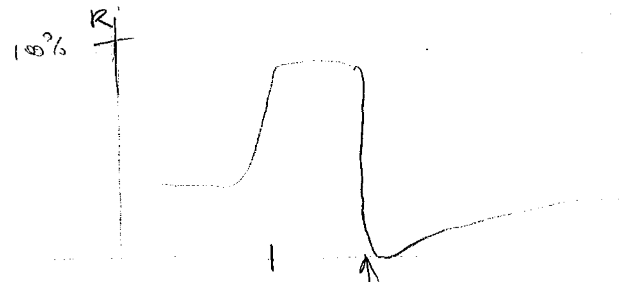


always for ϵ! → $$\boxed{\epsilon = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2} - i\frac{\omega\gamma}{\omega_T^2}}}$$

typical:
$\epsilon_0 = 15$
$\epsilon_\infty = 12$
$\frac{\gamma}{\omega_T} = .02$ (maybe)

→ can calculate $\tilde{n}$

→ can calculate R



handout Yu + Cardona pg 289, 290 graphs

freq at which $\epsilon = 0$
call it "ω_L"

Turns out... ω_T: frequency of TO phonons → regular case wave →
ω_L = " " LO phonons → strange case

⊖ ⊕ ⊖ ⊕ ↕ Electric field

obvious to see why this frequency can be absorbed / gets a response
See Stokes' quote next page

→ ← → ←
⊖ ⊕ ⊖ ⊕
wave →
↔
electric field.
can't really do that w/ external light wave
could happen internally, though

see next page also

Maxwell Eqn $\nabla\cdot\vec{D} = 0 \rightarrow \nabla\cdot(\epsilon\epsilon_0\vec{E}) = 0$

$\nabla\cdot\vec{E} \longrightarrow \vec{k}\cdot\vec{E}$

but $\vec{k}\cdot\vec{E}$ not zero for this case

so $\boxed{\epsilon_r = 0}$

pg 186

Stokes: "The response of the ions to the driving force of the electric field will be greatest when their motion is one of the natural lattice waves in the crystal. It is not difficult to identify the frequency at which this occurs. First of all, since the motion of the positive + negative ions is in opposite directions, the lattice wave belongs to the <u>optical</u> branch of the dispersion curves... Second, since the displacement of the ions is $\perp$ to the direction of the wave vector, the lattice wave is <u>transverse</u>."

Also: wavelengths must match $\rightarrow$ $k_{light} = k_{lattice\ wave}$

k_{light} $\downarrow$ essentially 0, as discussed before

$\Downarrow$ Therefore <u>zone center</u>

$\therefore$ ω_T = freq of TO at ~~zone~~ center!

(That's IR light — for KBr $\simeq$ 90 μm)

Stokes also has explanation of LO on pg 187

$\nabla \cdot \vec{D} = 0$

$\rightarrow \epsilon_r \epsilon_0 \, \vec{k} \cdot \vec{E} = 0$

can be satisfied if $\vec{k} \cdot \vec{E} = 0$ (usual!) transverse

<u>or</u> if $\epsilon_r = 0$

$\hookrightarrow$ then a longitudinal wave can be supported

Lyddane-Sachs-Teller

back to ~~Sellmeier~~ Lorentz permittivity eqn, but w/ $\gamma = 0$

$$\epsilon = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}}$$

when $\epsilon = 0$, $\omega = \omega_L$

$$0 = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 - \underbrace{\frac{\omega_L^2}{\omega_T^2}}_{x}}$$

$$0 = A + \frac{B - A}{1 - x}$$

$$0 = \cancel{A} - Ax + B - \cancel{A}$$

$$x = \frac{B}{A}$$

$$\boxed{\left(\frac{\omega_L}{\omega_T}\right)^2 = \frac{\epsilon_0}{\epsilon_\infty}} \quad \text{LST}$$

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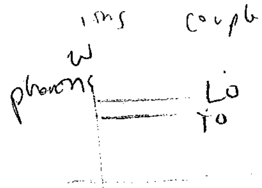
Polaritons - coupling of em wave w/ something

phonon polaritons (the only type mentioned in Kittel)

$\oplus \overset{\leftrightarrow}{} \ominus$ optical phonons
ions couple to EM wave.



photons (not quite vertical!)

$w = vk = \left(\frac{c}{n}\right)k$

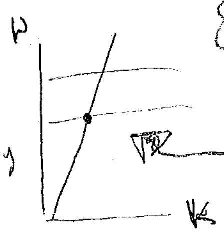
In regions where frequencies are degenerate (w_T),
really need degenerate pert. theory (like we did for band structure when two electron modes had same frequency).

Result → two coupled modes, "upper branch" + "lower branch", gap between them

At frequencies farther away, looks regular again.

Instead of QM degenerate pert. theory, use this method which gives essentially correct result

Kittel Fig 14.11 + 14.12 pg 411, 412



~~no traveling wave, k is pure imaginary~~

relevant area around $w_{TO} = \left(\frac{c}{n}\right)k$

Force dielectric constants to be the same

Light: $\frac{w}{k} = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon}} \rightarrow \varepsilon = \left(\frac{ck}{w}\right)^2$

TO phonons: $\varepsilon = \varepsilon_\infty + \dfrac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{w^2}{w_T^2}}$ (no damping, for simplicity)

$$\boxed{\frac{c^2k^2}{\omega^2} = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}}}$$

polariton dispersion relation

can in theory solve ω vs k

$\times \omega^2$

$$c^2k^2 = \varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{1 + \frac{\omega^2}{\omega_T^2}}$$

Hard to solve for ω,
Easier for k

$$\boxed{k = \frac{1}{c}\sqrt{\varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{1 + \frac{\omega^2}{\omega_T^2}}}}$$

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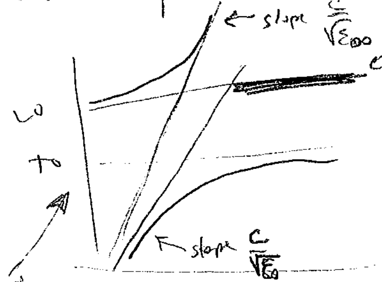
For a given ω, I can tell you k!



No (real) soln for k in here.

k is purely imaginary for those ω's

That's why it's very reflective for those freqs.

Consider ~~the~~ lower branch, small k / small ω

$$k = \frac{1}{c}\sqrt{\varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{1 + \cancel{\text{small}}}}$$

$$\underline{k = \frac{1}{c}\omega\sqrt{\varepsilon_0}}$$

Consider upper branch, large k/ω

$$k = \frac{1}{c}\sqrt{\varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{\cancel{1} + \frac{\omega^2}{\omega_T^2}}}$$

$$\sqrt{\varepsilon_\infty \omega^2 + \underbrace{\frac{\cancel{\omega^2}(\varepsilon_0 - \varepsilon_\infty)}{\cancel{\omega^2}/\omega_T^2}}_{\text{then this small}}}$$

$$\underline{k = \frac{1}{c}\omega\sqrt{\varepsilon_\infty}}$$