$$\boxed{\frac{c^2k^2}{\omega^2} = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}}}$$

polariton dispersion relation

can in theory solve ω vs k

$\times \omega^2$

$$c^2k^2 = \varepsilon_\infty \omega^2 + \frac{\omega^2(\varepsilon_0 - \varepsilon_\infty)}{1 + \frac{\omega^2}{\omega_T^2}}$$

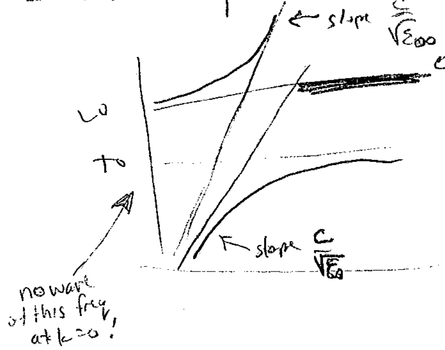
Hard to solve for ω.

Easier for k

$$\boxed{k = \frac{\omega}{c}\sqrt{\varepsilon_\infty \cancel{} + \frac{\cancel{}(\varepsilon_0 - \varepsilon_\infty)}{1 + \frac{\omega^2}{\omega_T^2}}}}$$

day 41 pg 1

For a given ω, I can tell you k!



at some pt, get these

← No (real) soln for k in here.

k is purely imaginary for those ω's

That's why it's very reflective for those freqs.

Consider ~~the~~ lower branch, small k / small ω

$$k = \frac{\omega}{c}\sqrt{\varepsilon_\infty \cancel{} + \frac{\cancel{}(\varepsilon_0 - \varepsilon_\infty)}{1 + \cancel{\text{small}}}}$$

$$\underline{k = \frac{1}{c}\,\omega\sqrt{\varepsilon_0}}$$

Consider upper branch, large k/ω

$$k = \frac{\omega}{c}\sqrt{\varepsilon_\infty \cancel{} + \frac{\cancel{}(\varepsilon_0 - \varepsilon_\infty)}{\cancel{1} + \frac{\omega^2}{\omega_T^2}}}$$

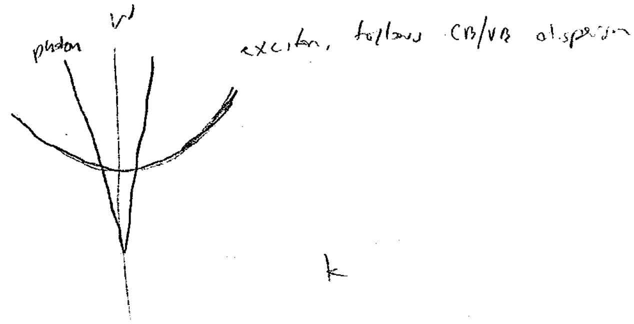
$$\sqrt{\varepsilon_\infty \omega^2 + \underbrace{\frac{\cancel{\omega^2}(\varepsilon_0 - \varepsilon_\infty)}{\cancel{\omega^2}/\omega_T^2}}_{\text{then this small}}}$$

$$\underline{k = \frac{1}{c}\,\omega\sqrt{\varepsilon_\infty}}$$

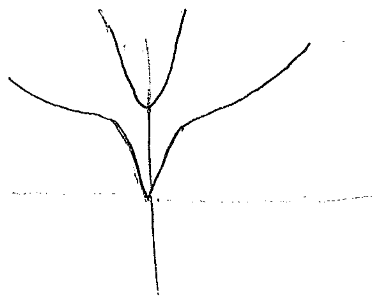
TOPS 35500

Exciton polariton

Do excitons interact w/ EM wave? of course!



result



Plasmons
back to
- conductors

$$\varepsilon = 1 + \underbrace{\frac{ne^2}{m\epsilon_0}}_{\omega_p^2} \underbrace{\frac{1}{\omega_0^2 - \omega^2}}_{\text{No restoring force now! No resonant freq!}}$$

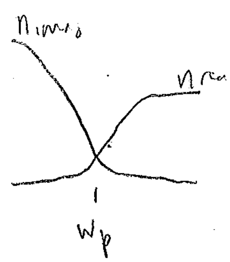
$$\boxed{\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}}$$

or really $\boxed{\varepsilon = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}}$ $= \varepsilon_\infty\left(1 - \frac{\omega_p^2/\varepsilon_\infty}{\omega_p^2}\right)$ $= \varepsilon_\infty\left(1 - \frac{\omega_{p\,\text{effective}}^2}{\omega^2}\right)$

if "positive ion core background has dielectric constant ε_∞ essentially constant up to frequencies well above ω_p"

Like w/ LO phonon, this has a longitudinal resonance when $\varepsilon = 0$
$\rightarrow \omega = \omega_p$

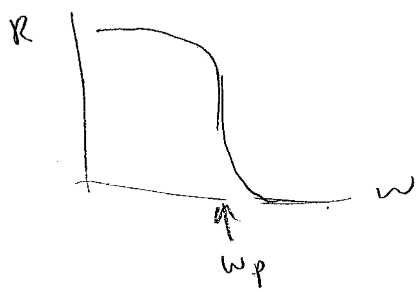
n & W plots for $\gamma = \omega_p/50$



when $\omega < \omega_p$
$\varepsilon < 0$

$n = \sqrt{\varepsilon}$ = purely imaginary!

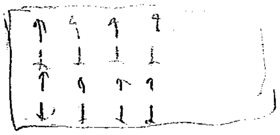
then $R = \left|\frac{1-\tilde{n}}{1+\tilde{n}}\right|^2 = 100\%$ (can show)



Now - mention

At ω_p, $\varepsilon = 0$, $\rightarrow$ longitudinal resonance.

pg 400 Kittel fig 14.4

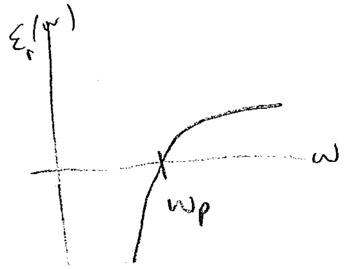


arrows = displacement of electrons

"plasmon" - a quantized plasma oscillation
come in units of $\hbar\omega_p$

day 41 pg 4

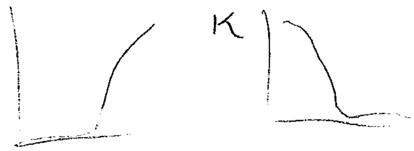
plasmons - free electrons in metal



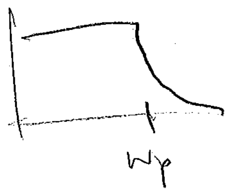
Kittel 394

P+W pg 54

→ n =



→ R =



Stokes pg 189 "conduction electrons"

problem Stokes 16-17

Calculate plasma freq for sodium $n = 2\cancel{.52} \cdot 10^{28} m^{-3}$ 2.65   Kittel Table 6.1

$$\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}}$$

$$= \sqrt{\frac{2.65 \cdot 10^{28} (1.6 \cdot 10^{-19})^2}{9.11 \cdot 10^{-31} \cdot 8.85 \cdot 10^{-12}}}$$

$$= \underline{9 \cdot 10^{15}/s} \text{ ish}$$

$$\lambda = \frac{c}{\omega/2\pi} = \cancel{2.56} \quad \underline{200\text{ nm}} \text{ ish}$$

Make plot of n vs ω vs γ = ~
K vs ω
R vs ω

with damping included

$$\varepsilon = 1 + \frac{\omega_p^2}{-\omega^2 - i\omega\gamma}$$

can skip

$$= 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \times \frac{\omega^2 - i\omega\gamma}{\omega^2 - i\omega\gamma}$$

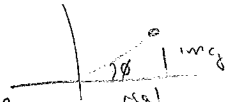
$$= 1 - \frac{\omega_p^2(\omega^2 - i\omega\gamma)}{\omega^4 + \omega^2\gamma^2}$$

$$(\varepsilon)_{real} = 1 - \frac{\omega_p^2\omega^2}{\omega^4 + \omega^2\gamma^2} = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2}$$

$$(\varepsilon)_{imag} = 1 + \frac{\omega_p^2\omega\gamma}{\omega^4 + \omega^2\gamma^2} = 1 + \frac{\omega_p^2\gamma}{\omega(\omega^2 + \gamma^2)}$$

same as Drude model

Could write ε in polar form



$$\tilde{n} = (\varepsilon)^{1/2} = \sqrt{|\varepsilon|}\, e^{i\phi/2}$$

$$R = \left|\frac{1 - \tilde{n}}{1 + \tilde{n}}\right|^2$$

problem → γ not constant w/ ω.

HW problem: just let γ = 0

What about dispersion curve ω vs k? Like polariton = comb of something w/ EM wave

again use $\frac{\omega}{k} = \frac{c}{n} \rightarrow \varepsilon = n^2 = \left(\frac{ck}{\omega}\right)^2$

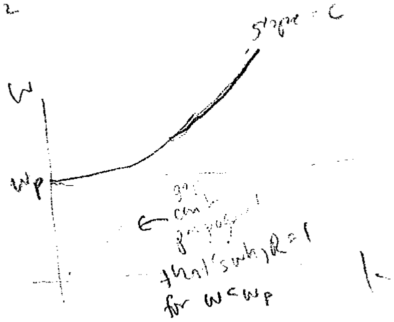
Force the two epsilons to be equal, Ignore γ

$$1 + \frac{\omega_p^2}{-\omega^2} = \frac{c^2k^2}{\omega^2}$$

(Solve for ω not hard $\omega = \sqrt{c^2k^2 + \omega_p^2}$) ← $k = \frac{1}{c}\sqrt{\omega^2 - \omega_p^2}$

large ω: (large k) $k = \frac{1}{c}\omega$ $\frac{\omega}{k} = c$

small k: $\omega^2 - \omega_p^2 \simeq 0$ $\omega = \omega_p$



← can't propagate that's why R=1 for ω<ωp

Stokes pg 190
Fig 16-11

Kittel Fig 14.2
pg 399