

Ionic



Attraction

$$-\frac{q^2}{P_{ij}R}$$
 for some P_{ij}

(SI: $\frac{1}{4\pi\epsilon_0} \frac{q^2}{r}$)

CGS: $\frac{q^2}{r}$

Repulsion

$$+\frac{q^2}{P_{ij}R}$$
 for some other P_{ij}

$$\lambda e^{-R/\rho}$$
 for Pauli exclusion (nearest neighbors)

$\lambda, \rho =$ empirical parameters

(reference = negative charge)

$$U_{tot} = N \left(2 \lambda e^{-R/\rho} - \sum_i \frac{(\pm) q^2}{P_{ij} R} \right)$$

Why not $N/2$?

because $N = \# \text{ pairs}$,
 i.e. $\# \text{ ions} = 2N$

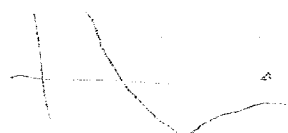
$$-\frac{q^2}{R} \sum_i \frac{(\pm)}{P_{ij}}$$

" " = α ; Madelung constant

calculate very similar to VdW stuff in last section

NaCl structure	$\alpha = 1.747565$
CsCl	$\alpha = 1.762675$
ZnS	$\alpha = 1.6381$

$$U_{tot} = N \left(2 \lambda e^{-R/\rho} - \frac{\alpha q^2}{R} \right)$$



same deal $\frac{\partial U}{\partial R} = 0$

guess R_0 (has a derivative eqn, numerical only)

then $U_{tot}(R=R_0) \rightarrow$ total lattice energy

has analytic:

$$U_{tot} = -\frac{N \alpha q^2}{R_0} \left(1 - \frac{\rho}{R_0} \right)$$

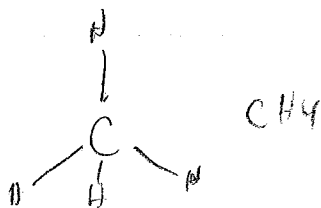
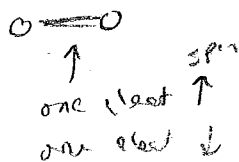
 "Madelung" ← repulsive

cohesive energy (a few eV, much larger than VdW noble gases)

table 7.966 nice agreement!

Covalent crystals

Ex: H₂



diamond structure (I can't draw)

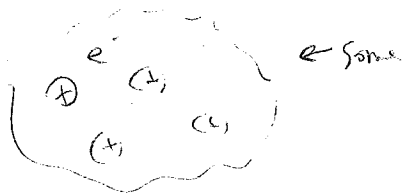
Number forbidden 222 diamond problem

comes from QM - orbitals (?)

Hard to analyze quantitatively from first principles

Metals

Some Electrons free (valence elect of atom → cond. electrons)



↓
but lowered in energy (due to quantum (?))

a weaker binding (ductile, malleable) → still crystalline on microscopic level
- close packed

↓ strong for transition metals though
Table 1 pg 50

↓ weaker than ionic
Compare kcal/mol to Table 7 pg 66

Hydrogen bonds

- already discussed
relatively

Atomic Radii

- basically, no such thing for universal case

distances between atoms depend strongly on structure

but can get approx

- ← vdw
- ← ionic (modification of vdw)
- ← covalent
- ← metallic

Table 9 pg 71
Table 10 pg 72
that's all I'll say

Elastic Strain

From Physics 121

$$\text{stress} = \text{Force/area}$$

$$\text{strain} = \Delta L/L$$

$$\text{Young's modulus } Y = \text{stress/strain}$$

Crystals: difference between eg [100] and [111] directions complicates things!

Goals: to find generalization of Y , called "stiffness constants" (tensor)

- For ~~simple~~ cubic crystals, get wave eqn for long wavelength waves
 → properties like wave speed (transverse + long.)
 in $\left. \begin{array}{l} [100] \\ [110] \\ [111] \text{ (for hcp)} \end{array} \right\} \begin{array}{l} \text{directions in terms} \\ \text{of stiffness} \\ \text{constants} \end{array}$

Vibrational Modes in a Solid

molecules 3 translational d.f
 2-3 rotational d.f
 ? vibrational d.f

Looking ahead

Solid \Rightarrow can use translational symmetry to simplify vibrational problems

- longitudinal vs transverse

- long wavelengths
 (solid like homogeneous medium)
 \downarrow but not isotropic
 rest of this chapter

vs. short wavelengths
 (must consider microscopic structure)
 \downarrow
 Next two chapters

Reminder: matrix multiplication. Change a vector into 3 new vector components

$$\begin{pmatrix} a_1' \\ a_2' \\ a_3' \end{pmatrix} = \begin{pmatrix} b \\ \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$a_j' = \sum_{k=1}^3 b_{jk} a_k$$

$$a_i' = \sum_{j=1}^3 b_{ij} a_j$$

To turn a matrix into a new matrix components?

$$b_{jk}' = \sum_l \sum_m c_{jkm} b_{lm}$$

b_{jk} = "rank 2 tensor" \rightarrow 9 components

c_{jkm} = "rank 4 tensor" \rightarrow 81 components