Brigham Young University
Department of Physics and Astronomy

Physics 601, Hirschmann — Fall 2011

Midterm

This is a 3 hour exam. You may use your text (Arfken), your class notes and your returned homework. You may not use any other sources such as other texts, the homework solution sets or Maple, etc. Choose four of the following five problems. Each is worth 25 points. Show all your work.

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1. **(a)** Prove the identity

$$\vec{u} \times \left(\vec{v} \times \vec{w}\right) + \vec{v} \times \left(\vec{w} \times \vec{u}\right) + \vec{w} \times \left(\vec{u} \times \vec{v}\right) = 0$$

(b) Show that for any *arbitrary* vector field $\vec{u}$ and scalar field ϕ

$$\oint_S \vec{u} \times \vec{\nabla}\phi \ \cdot \hat{n}\, da = \int_V \vec{\nabla}\phi \cdot (\vec{\nabla} \times \vec{u})\; dv$$

where V is an arbitrary volume bounded by the surface S.

2. Show that the following function is regular (finite) as $x \to 0$ and find the first nonzero term in the expansion

$$f(x) = \left(\frac{15}{x^4} - \frac{6}{x^2}\right)\sin x - \left(\frac{15}{x^3} - \frac{1}{x}\right)\cos x$$

3. Find the Laurent expansions around $z = i$ and $z = 1$ of

$$f(z) = \frac{1}{z^2 + 1}$$

and determine the regions in the complex plane where they are valid.

4. Determine the convergence or divergence of the following infinite series (p and a are constants)

$$(\mathbf{a}) \quad \sum_{n=2}^{\infty} \frac{(-1)^n\, n}{\sqrt{n^3 - 1}} \qquad\qquad (\mathbf{b}) \quad \sum_{n=1}^{\infty} \frac{1}{n^p\, a^n}$$

(c) Find allowable values of k, p, a and b so that the series below converges. k is an even integer (≥ 0), p is an odd integer (≥ -1), and a and b are positive, real numbers.

$$\sum_{n=0}^{\infty} \frac{(2n+p)!!}{(2n+k)!!}\,(a\,n + b)$$

5. **(a)** Find the imaginary part, $v(x, y)$ of an analytic function, $f(z)$, if its real part is given by

$$u(x, y) = e^x\,(y \cos y + x \sin y)$$

(b) For any second or third order polynomial, $p(z)$, with simple zeros in the complex plane, show

$$\oint_C \frac{1}{p(z)}\, dz = 0$$

where C encloses all the zeros of $p(z)$.